1. Use the matlab m-file on the course website `erwater.m` to compute the relative permittivity (also called dielectric constant) of pure water.

   (a) Plot the real part of the dielectric constant versus frequency at 275 and at 300 K.
      i. For $T = 275$ K, at about what frequency does the real part of the dielectric constant fall below 40?
      ii. How does the real part change with temperature for $f < 4$ GHz?
      iii. How does it change with temperature for $f > 4$ GHz?

   (b) Plot the imaginary part of relative permittivity versus frequency at 275 and 300 K.
      i. How does the frequency at which the imaginary part of the dielectric constant is greatest change with temperature?
      ii. At about what frequency does the imaginary part change the most with temperature?

   (c) Plot the real and imaginary parts of the refractive index versus frequency at 300 K.
      i. Does the imaginary part of the refractive index peak at the same frequency as the imaginary part of the dielectric constant?
      ii. Given your answer to Question 1(c)i, at what frequency is the attenuation of microwave radiation the strongest at $T = 300$ K?

   Use a frequency range of 100 MHz to 100 GHz for Questions 1a and 1b, and a frequency range of 1 GHz to 1 THz for Question 1c. A log scale will work the best.

2. (a) Plot the drop size distribution for a light rain (5 mm per hour) and heavy rain (40 mm per hour) using the Marshall–Palmer drop size distribution function. The Marshall–Palmer distribution is described by (5.116) in UMF. Although not mentioned in the text, the $b$ parameter is related to the expected value of the drop size diameter $\langle d \rangle$ (the expected value can be thought of as the average or most common drop diameter) through the relationship $b = \frac{1}{\langle d \rangle}$.

   (b) Find the total number of rain drops per cubic meter for the light and heavy rain.

   (c) The Rayleigh scattering approximation can be used when $|n_s k_o d| < 0.5$, where $n_s$ is the refractive index of the particles (assuming the background medium is air), $k_o$ is the free–space wavenumber, and $d$ is the diameter of the particle. Use the expected value of raindrop diameter, $\langle d \rangle$, in your calculations.
i. If the temperature of the rain drops are 275 K (near freezing), for what frequencies can the Rayleigh approximation be used for the light rain?

ii. For the heavy rain?

iii. Does your answer change significantly for warmer temperatures (around 290 K)?

3. (a) Find an expression for the absorption and scattering coefficients for rain, \( \kappa_a \) and \( \kappa_s \), assuming rain drops can be approximated as spherical particles and assuming Rayleigh scattering. Use scattering cross sections given by (5.75) and (5.76) in UMF. To find the coefficients, integrate the product of the drop size distribution function and either the absorption of scattering cross sections, \( \sigma_a \) and \( \sigma_s \), over all possible diameters.

\[
\kappa = \int_0^\infty p(a) \sigma(a) \, da
\]

where \( \kappa \) is either the absorption or scattering coefficient, \( p(a) \) is the drop size distribution, \( \sigma(a) \) is the cross section for absorption or scattering, and \( a \) is either the radius or diameter of the particles. Note that the absorption and scattering cross sections are called \( Q_a \) and \( Q_s \) in UMF. Write your answer in terms of the gamma function, \( \Gamma(x) \), where

\[
\Gamma(x) = \int_0^\infty t^{x-1}e^{-t} \, dt.
\]

Use either MATLAB or lookup tables to find the value of \( \Gamma(x) \). Check your calculations by comparing with Figure 5.26 in UMF. Note that Figure 5.26 was created using a different drop–size distribution (the Laws–Parsons as opposed to the Marshall–Palmer) so that your values will not be exactly the same as what you see in the figure, but they should be within a factor of 10. Why the discrepancy? Are all 25 mm rain rate (for example) rain events the same, and the same in different parts of the world? I don’t think so, and these models are definitely empirical (found to fit a specific data set). In addition, these descrepancies will depend on the wavelength.

(b) What is the the extinction coefficient, \( \kappa_e = \kappa_a + \kappa_s \), and the scattering albedo, \( \omega = \kappa_s/\kappa_e \), for a rain rate of 5 mm per hour assuming a temperature of 280 K at 10 GHz?

(c) What is the the extinction coefficient and the scattering albedo for a rain rate of 25 mm per hour assuming a temperature of 280 K at 10 GHz?

(d) What is the the extinction coefficient and the scattering albedo for a rain rate of 25 mm per hour assuming a temperature of 280 K at 85 GHz?

4. Use the m–file on the course website scat_layer.m to explore the radiometric characteristics of a scattering layer. To properly use this function, look at Figure 1 and read the comments at the beginning of the file or type help scat_layer at the MATLAB command prompt. The reason for the weird format of the output is because of the numerical technique that is used to solve the equations.

Example: for height = 100 m, \( f = 37 e 9 \) Hz, \( T = 275 \) K, \( \kappa_a = 1.7 \times 10^{-4} \) Np m\(^{-1}\) and \( \kappa_s = 9.6 \times 10^{-6} \) Np m\(^{-1}\) (I made these up, they do not correspond to a specific rain rate of which I am aware), \( T_{bo} = [0 \ 0 \ 0 \ 0 \ 0 \ 0] \), and \( z = 0 \), the result \( T_b = [0 \ 0 \ 0 \ 18.8966 \ 6.9841 \ 4.9719] \) is found, which corresponds to upwelling brightness temperatures at \( z = 0 \) m of zero kelvin, and downwelling brightness temperatures of approximately 19 K at 103\(^{\circ}\), 7 K at 131\(^{\circ}\), and 5 K at 159\(^{\circ}\) at \( z = 0 \) where 0\(^{\circ}\) is defined as straight up (+\( \hat{z} \) direction). For the same situation and boundary conditions, the result for \( z = 100 \) is \( T_b = [18.8966 \ 6.9841 \ 4.9719 \ 0 \ 0 \ 0] \)
Figure 1: Scattering layer for Problem 4.

Answer the following questions.

(a) If all boundary conditions are 0 K and the temperature of the scattering layer is also 0 K, what are the downwelling brightness temperatures at $z = 0$ m and the upwelling brightness temperatures at $z = 100$ m if $f = 37$ GHz and the rain rate is 25 mm per hour? Explain why this makes sense.

(b) If all boundary conditions are 275 K and the temperature of the scattering layer is also 275 K, what are the downwelling brightness temperatures at $z = 0$ m and the upwelling brightness temperatures at $z = 100$ m if $f = 37$ GHz and the rain rate is 25 mm per hour? Explain why this makes sense.

(c) If all boundary conditions are 0 K and the temperature of the scattering layer is 275 K, what are the downwelling brightness temperatures at $z = 0$ m and the upwelling brightness temperatures at $z = 100$ m if $f = 37$ GHz and the rain rate is 25 mm per hour? Explain the trend in brightness temperatures at both $z = 0$ and 100 m.

(d) Simulate rain over a prairie grassland using the following conditions: scattering layer height of 2 km; temperature of rain is 275 K; downwelling brightness temperatures of 60 K at all angles at the top of the scattering layer (atmospheric emission from above the clouds); and upwelling brightness temperatures of 290 K at all angles at the bottom of the scattering layer (approximately the brightness temperature of a grassland). For a frequency of 37 GHz and a rain rate of 25 mm per hour, report the upwelling brightness temperatures at 2 km and the downwelling brightness temperatures at the surface. Why is the change with angle at the top of this scattering layer different than for Problem 4c? (Hint: observe what happens when you try different boundary conditions). What is the
scattering albedo for the scattering layer?

(e) Use the same conditions in Problem 4d but change the frequency to 10 GHz. What is the new scattering albedo? Report the brightness temperatures at 2 km and at the surface and explain why the variation with angle at 2 km is different from Problem 4d.

(f) Use the same conditions in Problem 4d but change the rain rate to 5 mm per hour. What is the new scattering albedo? Report the brightness temperatures at 2 km and at the surface and explain why the variation with angle at 2 km again the same as for Problem 4d.

(g) Make up and answer your own problem by changing boundary conditions, frequency, rain rate, height of the scattering layer, etc.

5. Read Chapter 2 up to Section 2.2 (pages 18 to 48) of “Spectrum Management for Science in the 21st Century.”

(a) What fraction of the U.S. GDP (gross domestic product) is “sensitive” to weather and climate?

(b) Describe in your own words the “fundamental asymmetry between the spectral requirements of active communications services and passive environmental uses.”

(c) According to the report, what is the single most valuable data source that currently enables one–week weather forecasts?

(d) Why is the improvement in forecasts for the Southern Hemisphere, and not the Northern Hemisphere, highlighted in Figure 2.3?

(e) Why are microwave observations of atmospheric water vapor critical for understanding global climate change?

(f) How do microwave radiometers contribute to the measurement of sea surface height (ocean altimetry) made by microwave radars?

(g) Why is 19.35 GHz particularly well-suited for the detection of sea ice?