A flat plate is in orbit $6.4 \times 10^6$ m above Earth’s surface. The area of the plate is 1 m$^2$. The plane of the plate’s orbit is in the plane defined by Earth’s path around the sun. See Figure 1 for an illustration of Sun, Earth, and plate orbits. The phase of the plate’s orbit is defined as follows. When the plate crosses the Sun–Earth line the orbit phase is $\alpha = 0^\circ$. When the plate crosses the Sun–Earth line projected behind the earth, the orbit phase is $\alpha = 180^\circ$. See Figure 2. The unit vector normal to the surface of the plate always points to the center of the earth. Assume that the plate is a gray body (its emissivity is independent of wavelength) and that the plate’s surfaces are perfectly rough (the plate’s emissivity is the same at all wavelengths and in all directions). Assume that the plate’s thermal conductivity is so high that its temperature is approximately uniform (there are no temperature gradients across the plate). Assume that the plate’s thermal inertia is so low that it is always in thermal equilibrium with its radiation environment (the radiation absorbed by the plate is equal to the radiation emitted by the plate). Assume that the plate’s heat capacity is negligible. Assume that the sun is a black radiator (a perfect blackbody) at a temperature of 5900 K. To simplify the problem, assume that the earth is a black radiator at a temperature of 300 K. The radius of Earth is $6.4 \times 10^6$ m. The radius of the sun is $6.5 \times 10^8$ m. The Sun–Earth distance is $1.5 \times 10^{11}$ m.

1. Find an expression (not a single number) for the power radiated (emitted) by each side of the plate, which will be a function of the area of the plate, its emissivity, and its temperature.

2. Find an expression for the power emitted by Earth that is absorbed by the plate. This will be a function of the area of the plate, the absorptivity of the plate, and the temperature of Earth. Note that the plate is one Earth radius above the surface of Earth.

3. Find an expression for the power emitted by the sun that is absorbed by the plate. This will be a function of the area of the plate, the absorptivity of the plate, the temperature of the sun, the Sun radius, the Sun–Earth distance, and the plate’s orbit phase.
4. Since the plate is in thermal equilibrium with its radiation environment, invoke the conservation of energy and solve for the temperature of the plate. Give an expression for the plate temperature as a function of its orbit phase about Earth, $\alpha$.

5. Plot the temperature of the plate (in kelvin) as a function of its orbit phase about Earth if the emissivity of the plate is unity ($e = 1$). Your plot should look like Figure 3. Then answer these questions.

(a) Why is the plate’s temperature constant when $150^\circ \leq \alpha \leq 210^\circ$?

(b) What is the temperature of the plate at $\alpha = 180^\circ$, and why is the temperature of the plate the same at $\alpha = 90^\circ$, $\alpha = 270^\circ$, and $150^\circ \leq \alpha \leq 210^\circ$?

(c) What would the temperature of the plate be at $\alpha = 180^\circ$ if Earth were transparent?

(d) How would this plot change if the plate were orbiting Mars (at the same altitude of $6.4 \times 10^6$ m above Mars) instead of Earth? Report at least two differences.

6. Plot plate temperature versus orbit phase $\alpha$ to show how the temperature of the plate varies with orbit phase if the emissivity of the plate is 0.9. Use the expression for plate temperature derived in 5. Compare this plot with Figure 3 and explain how they are similar and how they are different.

7. Consider a “greenhouse” condition where the plate is black at short wavelengths (absorbs all radiation from the sun) but “gray” with an emissivity of 0.9 at long wavelengths (the wavelengths emitted by Earth and the plate). Plot the temperature of the plate as a function of the phase of the plate’s orbit around Earth. Explain why this plot makes sense considering the new emissivity of the plate.


(a) What would the emissivity of an “ideal black material” be?

(b) What would the absorptivity of an “ideal black material” be?

(c) What would the reflectivity of an “ideal black material” be?

(d) Explain how these researchers modified the surface of this material to make it “black.”

(e) Give at least two examples of how such an ideal black material could be used.

(f) Note the section at the end of the article titled “Some Potential Uses of Nanotechnologies.” Explain why it would be useful to have “thermo–chromic glass” regulate the transmissivity of a window pane (item 9).
Figure 1: Geometry of Sun, Earth, and plate orbits. All orbits are in the plane of the page. Not drawn to scale.

Figure 2: Geometry of the plate orbit around the earth. Not drawn to scale.
Figure 3: Plate temperature as a function of orbit phase $\alpha$. 

\[ T_p = \text{plate thermal temperature, K} \]

\[ \alpha = \text{plate orbit phase, deg} \]