1. Find $\omega$, $k_0$, $\lambda_0$, $n$, $k$, $\lambda$, $u_p$, $\alpha = 2k''$ (the power attenuation constant), and $\delta_s$ if:

(a) $f = 500$ MHz and $\epsilon_r = 8$.
(b) $f = 500$ MHz and $\epsilon_r = 8 - j5$.
(c) $f = 6.9$ GHz and $\epsilon_r = 8 - j5$.
(d) $f = 6.9$ GHz and $\epsilon_r = 30 - j10$.

Remember that $n$ and $k$ are complex in general. Consider writing a short MATLAB code to verify your calculations. Consider organizing your answers in a table.

2. Given $\tilde{E} = \hat{x} \tilde{E}_x$ where $\tilde{E}_x = \tilde{E}_{xo} e^{jkz}$ (assume $\tilde{E}_{xo}$ is purely real):

(a) explain why $\tilde{E}$ propagates in the $-\hat{z}$ direction (hint: find the actual electric field by changing from the time–harmonic form and recall the form of a wave equation);
(b) show that $\tilde{E}$ solves the Helmholtz equation;
(c) find $\tilde{H}$;
(d) and show that $\tilde{H}$ solves the homogeneous wave equation $\nabla^2 \tilde{H} + k^2 \tilde{H} = 0$.

Note that $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2$ in Cartesian coordinates.

3. Use “Wave on a String” on the PHET (http://phet.colorado.edu) website to explore wave motion. This is a Shockwave Flash Object: open with a web browser that has the Adobe Flash Player installed. Play and experiment. Then do the following. Set to “No End.” Set to “Oscillate.”

(a) Find the wavelength $\lambda$ and the phase velocity $u_p$ for the settings shown in Figures 1, 2, and 3. Use the “Rulers,” “Timer,” and “draggable reference line” to help. Note that the values for “amplitude,” “frequency,” and “damping” are only relative and not absolute, which means that if you want to find the actual frequency (which may be helpful in determining the phase velocity), you will need to use the “Timer” and count the revolutions of the oscillator.
(b) What material (constitutive) electrical property is analogous to “tension” in the simulation?

(c) As “tension” is increased, does this material electrical property increase or decrease? Explain how you determined this.

(d) Set to “No End.” Set to “Oscillate.” Use the setting shown in Figure 4. What material electrical property (constitutive property) is analogous to “damping” in the simulation?

4. Consider the ocean surface to be the \(x-y\) plane at \(z = 0\) and that \(z\) increases as you travel farther away from the surface, deeper into the ocean. A uniform plane wave is propagating in the +\(\hat{z}\) direction. The constitutive parameters of sea water are \(\varepsilon_r = 82\), \(\mu_r = 1\), and the conductivity \(\sigma = 6 \text{ S m}^{-1}\). If the magnetic field at \(z = 0^+\) (just inside the ocean, so we are not concerned with surface effects) is

\[
\mathbf{H}(z = 0^+, t) = \hat{y} 150 \cos \left( 4\pi \times 10^3 t + \frac{\pi}{9} \right) \text{ mA m}^{-1} \tag{1}
\]

(a) find the time harmonic form of the magnetic field, \(\tilde{\mathbf{H}}(z)\), for \(z > 0\);

(b) find the time harmonic form of the electric field, \(\tilde{\mathbf{E}}(z)\), for \(z > 0\);

(c) find the real, time–dependent electric fields \(\mathbf{H}(z, t)\) and \(\mathbf{E}(z, t)\), for \(z > 0\);

(d) illustrate how \(\mathbf{E}(z, t)\) propagates in the \(\hat{\mathbf{z}}\) direction by generating a MATLAB figure (consider using the example m–file from the last problem set);

(e) determine at what depth the electric field is 5% of its value at \(z = 0^+\).

5. Use “Radio Waves” on the PHET website to explore wave motion. Play and experiment. Then answer the following.

(a) Even though the radiation propagating away from the radio station antenna is traveling through air (which can be approximated well by free–space), the magnitude of the field decreases. Why? Hint: think about energy conservation, a control volume around the transmitting antenna, and the surface area of this control volume as the control volume gets larger and larger.

(b) Considering your answer to 5a, what is one limitation of using a uniform plane wave approximation of a true spherical wave?

(a) What three types of remote sensing technologies will be used by the Solar Dynamics Observatory?
(b) What is the period of the “solar cycle” and what happens?
(c) What other country besides the US is sponsoring Aquarius?
(d) What type of remote sensing is used by Aquarius? (Not specified in the story, see link on course webpage.)
(e) What typically controls the changes of the salinity of ocean water?
(f) What type of remote sensing is used by Glory? (Again, not specified, do a web search.)
(g) Briefly describe how aerosols both warm and cool Earth.