When the soil is bare (no vegetation), Kirchhoff's law can be used to find the brightness temperature of the soil.

\[ T_b(\theta) = e_{\text{soil}}(\theta) \cdot T_{\text{soil}} \]

\[ e_{\text{soil}}(\theta) = 1 - R_{\text{soil}}(\theta) \]

\[ R_{\text{soil}}(\theta) = R_{\text{soil}}(\theta) \cdot e^{-\frac{h_{\text{soil}}}{\lambda}} \]

**Since** \( R_{\text{soil}} = f(\theta, h_{\text{soil}}) \)

**and since** \( h_{\text{soil}} = f(\theta_v, T_{\text{soil}}) \)

where \( \theta_v = \text{volumetric soil water content} \)

\[ \theta_v = \frac{\text{volume of liquid water}}{\text{volume of soil}} \]

\[ [\theta_v] = m^3 \cdot m^{-3} \]

Then, \( T_b(\theta) = f(\theta_v, T_{\text{soil}}) \)

**Roughness parameter**

The roughness parameter, \( h_{\text{soil}} \), accounts for the effect of roughness on the soil reflectivity and hence emissivity.

**Example roughness parameter** (Wigneron et al., 2001):

\[ h_{\text{soil}} = A \theta_v^B \left( \frac{\sigma}{\lambda} \right)^C \]

where \( A, B, \) and \( C \) are empirical constants that depend on frequency,

\( \sigma = \text{standard deviation of soil surface height} \quad 5 \text{mm} < \sigma < 30 \text{mm} \)

\( \lambda = \text{correlation length} \) (distance at which height has changed "significantly")

\( 50 \text{cm} < \lambda < 100 \text{cm} \)
We have so far considered only "perfect dielectrics" when considering the soil or ocean. A perfect dielectric is a material for which \( \text{Im} \mathbf{\gamma} = \sqrt{\varepsilon_r} = 0 \), i.e. no loss/absorption of radiation. Does a perfect dielectric emit radiation?

Consider radiative transfer:

\[
\frac{dT_b}{dT} = -k_e T_b + k_e T + \frac{k_e}{4\pi} \int T_b(\theta') d\Omega'
\]

If there is no absorption, how can there be emission without a change in temperature? There can't be!

Hence natural materials that emit radiation, like soil and ocean water, are not perfect dielectrics.

Since \( \text{Im} \mathbf{\gamma} \neq 0 \),

- Snell's laws still hold (still must satisfy the phase matching condition) but we must interpret them differently since \( n_1 \sin \Theta_1 = n_2 \sin \Theta_2 \) and what does it mean if \( \sin \Theta \) is complex? The angle of refraction is slightly different and there are evanescent waves along the surface.
- There is a layer of soil/water near the surface that contributes the majority of emitted radiation and we call this the emitting depth.
Layered Media

In reality a soil or ocean is not isothermal nor homogeneous. Temperature changes with distance into these real media. Since electrical properties depend on temperature, they also change with depth. More significantly, chemical gradients (water in soil, salinity in oceans) exist which also cause the electrical properties to change with depth. One way to deal with these more realistic situations is to use a model with multiple layers. E.g., what is the brightness temperature of an isothermal two-layer medium?

\[
\begin{align*}
T_x^+ (z=d^-_2, z) &= \text{known} = T_0 \\
T_x^+ (z=d^+_1, z) &= T_{x_1} + T_{x_2} (z=d^+_1, z) + \frac{R_{x_2}}{2} T_{x_2} (z=d^-_1, z) \\
T_x^- (z=0^+_1, z) &= \frac{R_{x_1}}{2} T_{x_1} (z=0^+_1, z) e^{-\frac{k_{a_2}}{2} z} + T_0 \left[ 1 - e^{-\frac{k_{a_2}}{2} z} \right] \\
T_x^- (z=0^+_2, z) &= \text{emission from dry soil layer} \\
T_x^- (z=d^+_2, z) &= T_0 \left[ 1 - e^{-\frac{k_{a_2}}{2} z} \right] + T_x^- (z=0^+_2, z) e^{-\frac{k_{a_2}}{2} z} \\
T_x^+ (z=0^+_1, z) &= \frac{R_{x_1}}{2} T_{x_1} (z=0^+_1, z) \\
\end{align*}
\]

5 equations

5 unknowns

See figure...
Emitting Depth

The layered medium example illustrated how there is a point at which a boundary between two layers with distinct electrical properties "can not be seen." The emitting depth is the layer near the surface that contributes the majority of the emitted radiation, and therefore is the layer to which a microwave radiometer is sensitive to changes in the properties of that layer. It is "how far" a radiometer can "see into" a surface.

\[
T_b^+(z=0, \mu_2) = T_b(z=-d, \mu_2) e^{-\frac{T(z=0)}{\mu_2}} + \int_0^{-d} k(z) T(z') e^{-\frac{T(z')}{\mu_2}} d\mu_2
\]

There is some depth \( z = -d \) at which the radiation is not a "significant" portion of the total radiation that reaches the surface \( \Rightarrow d \) would be the emitting depth.

The emitting depth depends on the optical depth \( T \):

\[
e^{-T} \Bigg|_{T=1} = e^{-1} = 37\% \quad \text{-- decrease in power level}
\]

\[
T = 2k''d \quad \text{When } T=1, \quad d = \frac{k''}{2k''} = \frac{1}{2k''} = \frac{1}{2\frac{\mu}{\omega}} = \frac{d_s}{4\pi n''} \quad \text{\( \Rightarrow \) the vertical depth of one optical depth}
\]

Note that:
\[
d = \lambda_0
\]
\[
d \approx \frac{1}{n''}
\]
For water @ 19.6 GHz (\( \lambda = 0.016 \text{ m} \)) \( d = \frac{0.016 \text{ m}}{4 \pi (2.5)} \equiv 0.5 \text{ mm} \)

\[ \text{C} \text{ 1.46 GHz} \ (\lambda = 21 \text{ cm}) \quad d = \frac{0.21 \text{ m}}{4 \pi (1.3)} \equiv 6 \text{ cm} \]

For wet soil @ 19.6 GHz \( d = \frac{0.016 \text{ m}}{4 \pi (1.3)} \equiv 1 \text{ mm} \)

\[ \text{C} \text{ 1.46 GHz} \quad d = \frac{0.21 \text{ m}}{4 \pi (0.1)} \approx 0.2 \text{ m} \] — actually not very realistic, accepted emitting depth = 5 cm

**Other considerations**

**Effect of temperature gradients?**

\[ T_{\text{soil}} = T_{\infty} + (T_{\text{surf}} - T_{\infty}) C \quad C = 0.25 \text{ for 1.46 GHz} \]

Choudhury et al., 19

\[ T_{\text{soil}} = T_{\infty} + (T_{\text{surf}} - T_{\infty}) (\Theta T / W_s)^B \]

\( W_s = 0.74 \), \( B = 0.758 \) for 1.46 GHz

Wigneron et al., 20

where \( T_{\infty} = \) deep soil (\(-50 \text{ cm}\)) temperature

\( T_{\text{surf}} = \) surface temperature (\(-1.5 \text{ cm}\))

**Effect of soil moisture gradient?**

Scattering in soil or ocean?
Fig. 5.26 Rain extinction coefficient $\kappa_v$ (Np km$^{-1}$) as a function of rain rate $R$, (Setzer, 1970).

$\text{temp}K$  \hspace{1cm} $\text{ Aim}$

\begin{align*}
[ks \ ka] &= \text{kappa_rain}(293, 0.05, 5) \\
ks &= 9.5419e-08 \\
ka &= 1.5202e-06 \\
ke &= ka + ks = 1.6156e-06 \\
[ks \ ka] &= \text{kappa_rain}(293, 0.10, 25) \\
ks &= 6.3561e-08 \\
ka &= 1.4698e-06 \\
ke &= ka + ks = 1.5334e-06 \\
diary off
\end{align*}

Setzer used the Laws-Parsons drop-size distribution, we are using Marshall-Palmer.

all units Np m$^{-1}$
Fig. E.47 Measured dielectric constant for five soils at 1.4 GHz.