2/09/2012

Group activity

What temperature is associated with the curve?

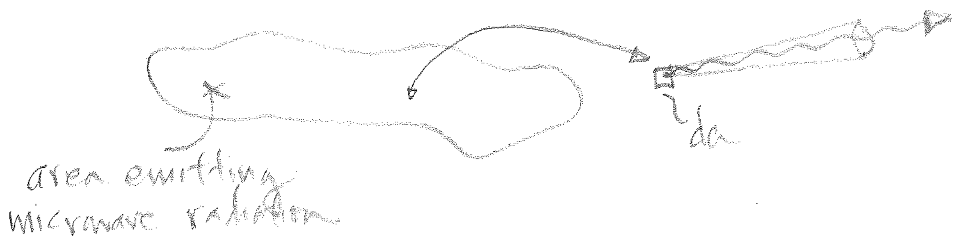
Wien's displacement law: $f_{max} = 5.87\times10^{10}\,T$, $T = \dfrac{f_{max}}{5.87\times10^{10}} = \dfrac{10^{13}\,\text{Hz}}{5.87\times10^{10}} \approx 170\,\text{K}$

Ask student to give Rayleigh-Jeans law: $B_f(f,T) = \dfrac{2k}{\lambda^2}T$

and give the units: $[B_f(f,T)] = W\cdot m^{-2}\cdot sr^{-1}\cdot Hz^{-1}$

We can use the Rayleigh-Jeans law instead of the full Planck law in microwave remote sensing at typical Earth surface temperatures with very little error.

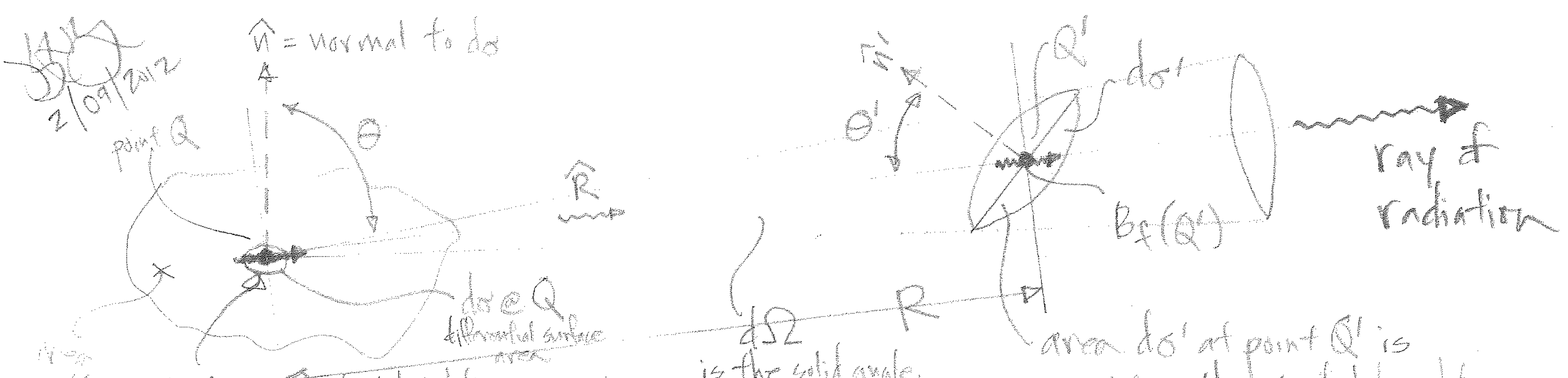
What is so great about spectral brightness?



$B_f(f,T,\theta,\phi)$ = spectral brightness @ frequency f, temperature T, and direction (θ,ϕ).

An important property of spectral brightness is that it is constant along any ray (path of propagation) in free-space!
In other words, spectral brightness is invariant with distance.
It doesn't "spread out" like power density (recall $S_R \sim 1/R^2$).
Therefor, any change in spectral brightness is due to the properties of the medium through which the radiation propagates! Media leave their "fingerprints" on spectral brightness and in remote sensing we use these fingerprints to identify the media and its properties.
Brightness ($W\cdot m^{-2}\cdot sr^{-1}$) is also invariant with distance.

J.H.K. 2/09/2012



$B_f(Q)$ = spectral brightness emitted at point Q

$d\Omega$ is the solid angle through which $B_f(Q)$ is emitted

area $d\sigma'$ at point Q' is receiving the spectral brightness $B_f(Q')$,

$$d\underline{s}' = \hat{n}' d\sigma'$$

$$[B_f(Q)] = \frac{W}{m^2 \cdot sr \cdot Hz} = \frac{J/s}{m^2 \cdot sr \cdot Hz}$$

$$d\Omega = \frac{\text{subtended area}}{R^2} = \frac{\hat{R} \cdot d\underline{s}'}{R^2} = \frac{\hat{R} \cdot \hat{n}' d\sigma'}{R^2} = \frac{\cos\theta' \, d\sigma'}{R^2}$$

where $\cos\theta' \, d\sigma'$ is the area "seen" by $d\sigma$ at point Q

energy emitted by $d\sigma$ at point Q

$$dE = \left[(B_f(Q)\hat{R}) \cdot d\underline{s} \right] d\Omega \, df \, dt, \qquad [dE] = J$$

$$= \left[B_f(Q)\hat{R} \cdot \hat{n} \, d\sigma \right] d\Omega \, df \, dt = B_f(Q) \underbrace{\cos\theta \, d\sigma}_{\text{area emitting}} \frac{\cos\theta' \, d\sigma'}{R^2} df \, dt = B_f(Q) \frac{\cos\theta \cos\theta' \, d\sigma \, d\sigma' \, df \, dt}{R^2}$$

reverse perspective

$$d\Omega' = \frac{\cos\theta \, d\sigma}{R^2}$$

energy incident on $d\sigma'$ at Q'

$$dE' = \left[(B_f(Q')\hat{R}) \cdot d\underline{s}' \right] d\Omega' \, df \, dt = B_f(Q')\hat{R} \cdot \hat{n}' d\sigma' \, d\Omega' \, df \, dt$$

$$= B_f(Q') \cos\theta' \, d\sigma' \frac{\cos\theta \, d\sigma}{R^2} df \, dt = B_f(Q') \frac{\cos\theta' \cos\theta \, d\sigma \, d\sigma' \, df \, dt}{R^2}$$

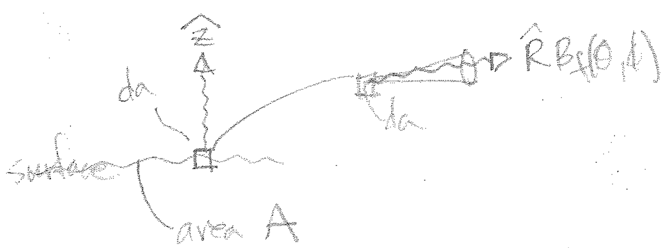
By conservation of energy, $dE = dE'$, so...

$$\boxed{B_f(Q) = B_f(Q')}$$

spectral brightness is invariant with distance!

2/09/2012

Total hemispherical radiation emitted by a blackbody surface



$$P_{rad} = \text{total power radiated by surface} = \int_A \int_{2\pi} \int_0^{\infty} B_f(\theta,\phi)\hat{R}\cdot d\vec{s}\, d\Omega\, df$$

ask

$$B \doteq \int_0^{\infty} B_f(f,T)\,df = \text{total brightness over all frequencies} = \cdots = \underbrace{\frac{\sigma T^4}{\pi}}_{\text{Stefan-Boltzmann law}} , \qquad [B] = W\cdot m^{-2}\cdot sr^{-1}$$

ask

but this doesn't look like the Stefan-Boltzmann law that we know!

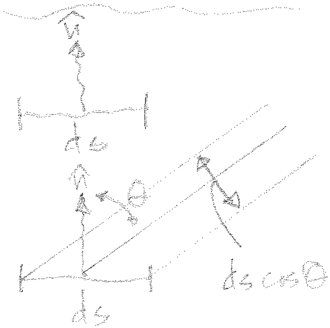
$\sigma = 5.673\times10^{-8}\ W\cdot m^{-2} K^{-4}\cdot sr^{-1}$

Find Prad

$$P_{rad} = \int_A \int_{2\pi} B(\theta,\phi)\hat{R}\cdot d\vec{s}\, d\Omega = \int_A \int_{2\pi} B(\theta,\phi)\hat{R}\cdot\hat{z}\,da\,d\Omega = \int_A \int_{2\pi} B(\theta,\phi)\hat{R}\cdot \overbrace{(\hat{R}\cos\theta - \hat{\theta}\sin\theta)}^{\hat{z}\text{ in spherical coordinates}} da\, d\Omega$$

$$= \int_A \int_{2\pi} B(\theta,\phi)\cos\theta\, da\, d\Omega$$

cosine law! effective area that radiates decreases with θ



effective area $\to 0$ as $\theta \to \frac{\pi}{2}$

$$P_{rad} = \int_A \int_{2\pi} B\cos\theta\, da\, d\Omega$$

blackbody brightness is isotropic, i.e. the same in every direction! (no dependence on θ and ϕ in the Planck law)

$$P_{rad} = \int_A \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{\sigma T^4}{\pi}\cos\theta\, da \sin\theta\, d\theta\, d\phi = \int_A \left\{ \frac{\sigma T^4}{\pi} 2\pi \int_0^{\frac{\pi}{2}} \cos\theta\sin\theta\, d\theta \right\} da$$

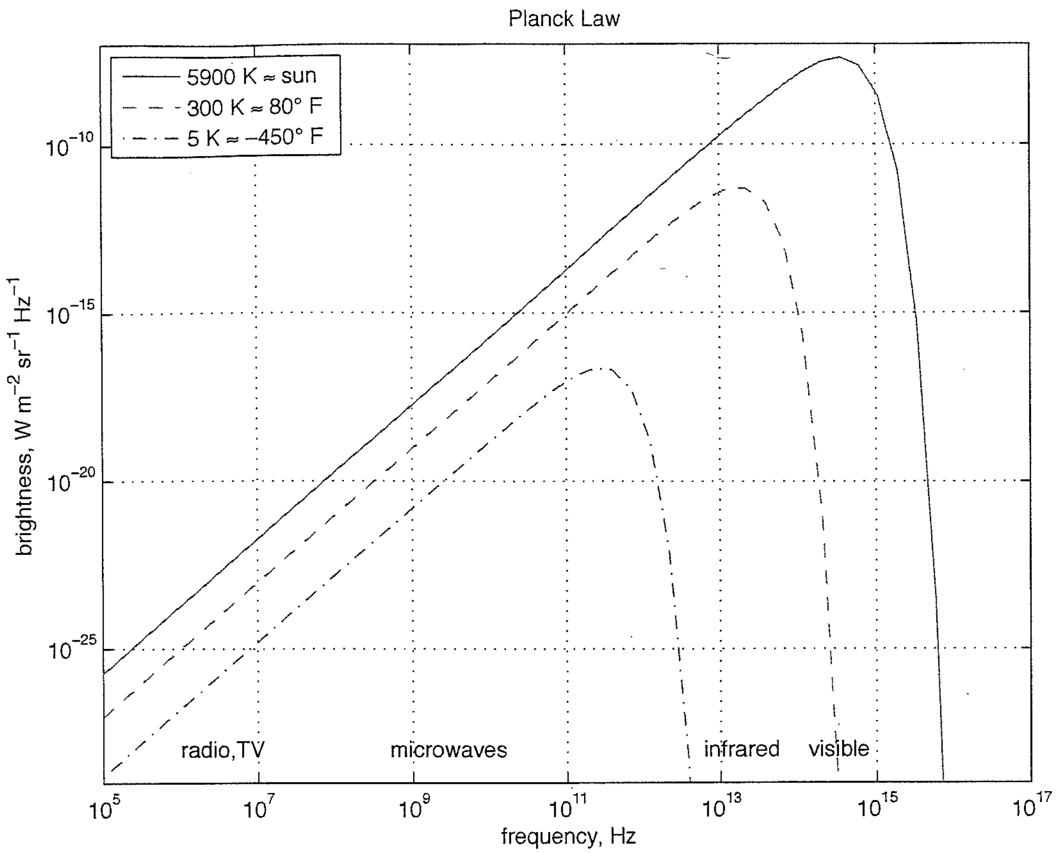
$$= \int_A \left\{ \frac{\sigma T^4}{\cancel{\pi}} \cancel{2\pi}\left(\frac{1}{2}\right) \right\} da = A\sigma T^4 = \text{power radiated by surface of area A}$$

$$\boxed{\sigma T^4 = \text{radiant emittance of a blackbody surface}}$$

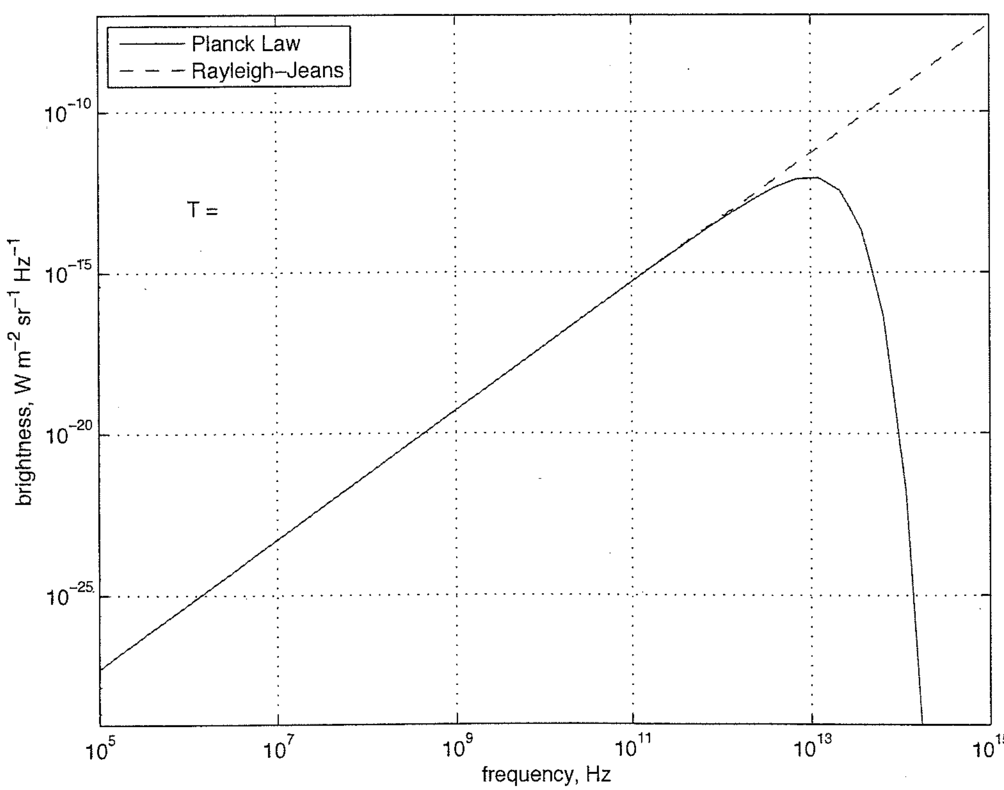
this is more familiar!

$[\sigma T^4] = W\cdot m^{-2}$

ask



Planck Law
5900 K ≈ sun
300 K ≈ 80° F
5 K ≈ −450° F
10^{-10}
10^{-15}
10^{-20}
10^{-25}
brightness, W m^{-2} sr^{-1} Hz^{-1}
radio,TV
microwaves
infrared
visible
10^{5}
10^{7}
10^{9}
10^{11}
10^{13}
10^{15}
10^{17}
frequency, Hz



Planck Law
Rayleigh–Jeans
T =
10^{-10}
10^{-15}
10^{-20}
10^{-25}
brightness, W m^{-2} sr^{-1} Hz^{-1}
10^{5}
10^{7}
10^{9}
10^{11}
10^{13}
10^{15}
frequency, Hz