Microwave Radiometry & Earth's Surface

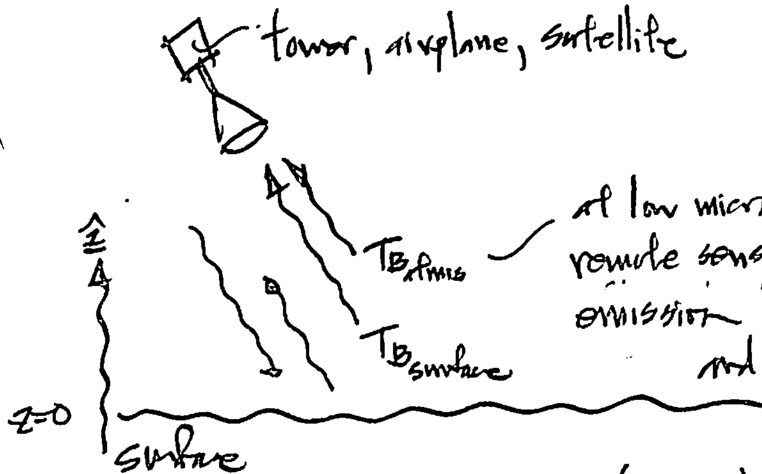
4/15/06

Recall 3 case examples:

① microwave radiometry of Earth atmosphere (emission only) → total precipitable water, temperature profile, water vapor profile (Aqua and TRMM)

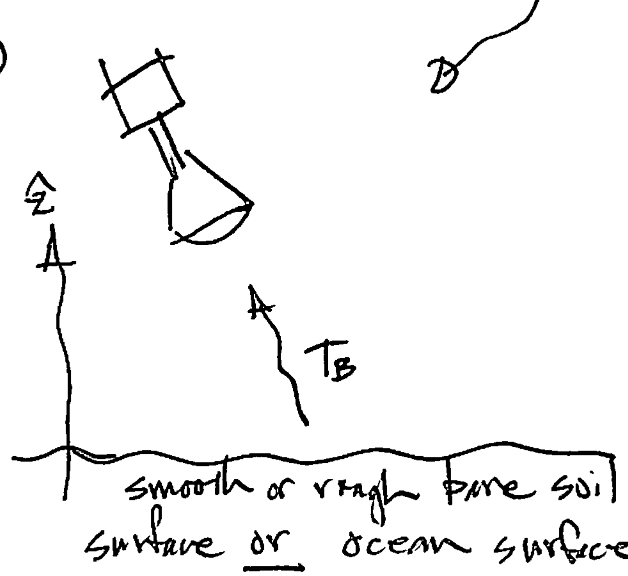
② microwave radiometry of Earth atmosphere (emission and scattering) → clouds, precipitation (rain and snow) (TRMM)

③ microwave radiometry of Earth surface → soil moisture, sea surface salinity, vegetation biomass (SMOS, the European Space Agency's Soil Moisture and Ocean Salinity mission, to be launched later this year)



at low microwave frequencies best for surface remote sensing, atmospheric emission and atmospheric emission reflected by the surface are both small and can be neglected

3a



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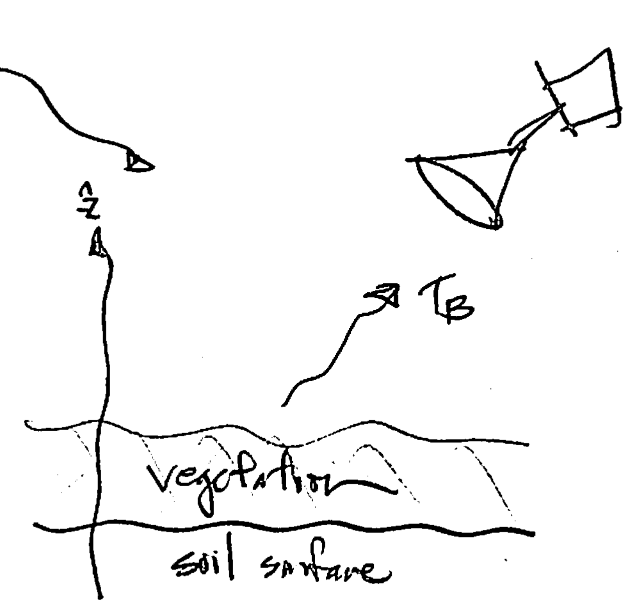


4/15/08

Reflection and Transmission

What happens to an electromagnetic wave when it encounters the boundary between two media with different electrical properties?

Must first make definitions:

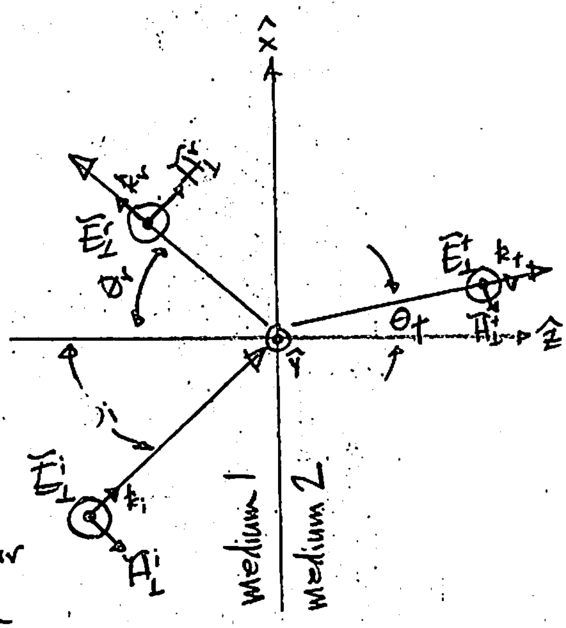
parallel polarization: wave whose electric field is parallel to the plane of incidence

perpendicular polarization: ... electric field perpendicular...

plane of incidence: plane containing the normal to the boundary and the direction of propagation of incident wave

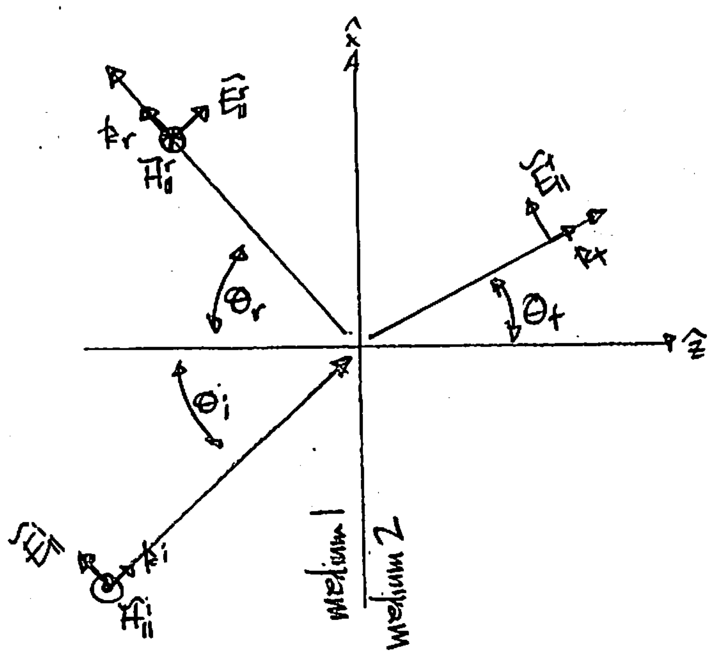
later relate to v-pol and h-pol

plane of incidence is the x-z plane



perpendicular polarization

parallel polarization



see figure 8-14 in handout

recall

$$\underline{H} = \frac{1}{\eta}\hat{k} \times \underline{E}$$

$$\underline{E} = -\eta\hat{k} \times \underline{H}$$

$\hat{k}$ = direction of propagation

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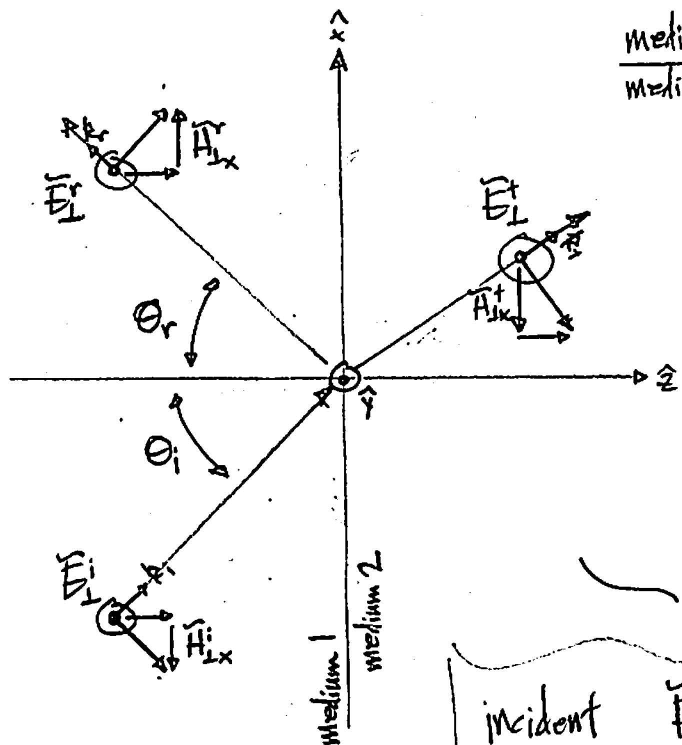
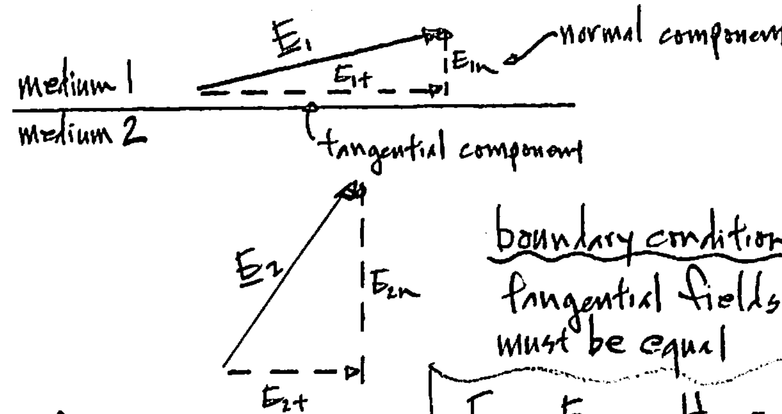
Examine perpendicular polarization

To find the relationship between incident, reflected, and transmitted fields, must use boundary conditions.





boundary condition

tangential fields must be equal

$$\boxed{E_{1t} = E_{2t}, \quad H_{1t} = H_{2t}}$$

reference: Chapter 8 of Ulaby's Fundamentals of Applied Electromagnetics

Section 2-6 of UMF

for some geometry,

incident wave

$$\tilde{E}_\perp^i = \hat{y}\, E_{\perp 0}^i\, e^{-jk_1(x \sin\theta_i + z\cos\theta_i)}$$

$$\tilde{H}_\perp^i = (-\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)\frac{E_{\perp 0}^i}{\eta_1} e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}$$

reflected wave

$$\tilde{E}_\perp^r = \hat{y}\, E_{\perp 0}^r\, e^{-jk_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\tilde{H}_\perp^r = (\hat{x}\cos\theta_r + \hat{z}\sin\theta_r)\frac{E_{\perp 0}^r}{\eta_1} e^{-jk_1(x\sin\theta_r - z\cos\theta_r)}$$

transmitted wave

$$\tilde{E}_\perp^t = \ldots$$

$$\tilde{H}_\perp^r = \ldots$$

boundary conditions apply to the total field

total field in medium 1

$$\tilde{E}^1 = \tilde{E}_\perp^i + \tilde{E}_\perp^r$$

$$\tilde{H}^1 = \tilde{H}_\perp^i + \tilde{H}_\perp^r$$

total field in medium 2

$$\tilde{E}^2 = \tilde{E}_\perp^t \qquad \tilde{H}^2 = \tilde{H}_\perp^t$$

apply boundary conditions: tangential fields must be equal!

BC's

$$\boxed{k_1 x \sin\theta_i = k_1 x \sin\theta_r = k_2 x \sin\theta_t}$$

"phase matching condition"

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phase matching condition

$$\underbrace{k_1 x \sin\theta_i = k_1 x \sin\theta_r = k_2 x \sin\theta_t}$$

$n = \sqrt{\epsilon_r}$ = index of refraction

recall $k\big|_{\mu_r=1} = k_0 n = \frac{2\pi}{\lambda_0} n$

$= \frac{2\pi}{c/f} n = 2\pi f \frac{n}{c}$

$= \omega \frac{\sqrt{\epsilon_r}}{\frac{1}{\sqrt{\mu_0\epsilon_0}}} = \omega\sqrt{\mu\epsilon}$

$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r\mu_0}{\epsilon_r\epsilon_0}}\Bigg|_{\mu_r=1} = \frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}}$

$= \frac{\eta_0}{n}$

$$\boxed{\theta_i = \theta_r}$$

Snell's law of reflection!

$n_1 = n_2,\ \theta_t = \theta_i$

$n_2 > n_1,\ \theta_t < \theta_i$

$n_1 > n_2,\ \theta_t > \theta_i$

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

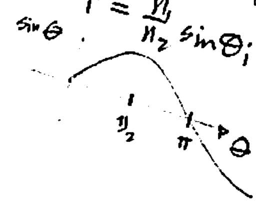
$$k_0 n_1 \sin\theta_i = k_0 n_2 \sin\theta_t$$

$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$$

Snell's law of refraction

use pencil to illustrate

$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$



more geometry...

$$\Gamma_\perp \triangleq \frac{E^r_{\perp 0}}{E^i_{\perp 0}}$$ = Fresnel reflection coefficient for perpendicular polarization = $\dfrac{\text{perpendicularly-polarized component of reflected electric field}}{\text{perpendicularly-polarized component of incident electric field}}$

$$= \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}\Bigg|_{\mu_r=1} = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t}$$

$$T_\perp \triangleq \frac{E^t_{\perp 0}}{E^i_{\perp 0}}$$ = Fresnel transmission coefficient for perpendicular polarization $= \cdots = \dfrac{2 n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t}$

$$T_\perp = 1 + \Gamma_\perp$$

ask

note these Fresnel coefficients are for perpendicularly-polarized electric fields — different expressions for magnetic fields and for parallel polarization!

4/17/08

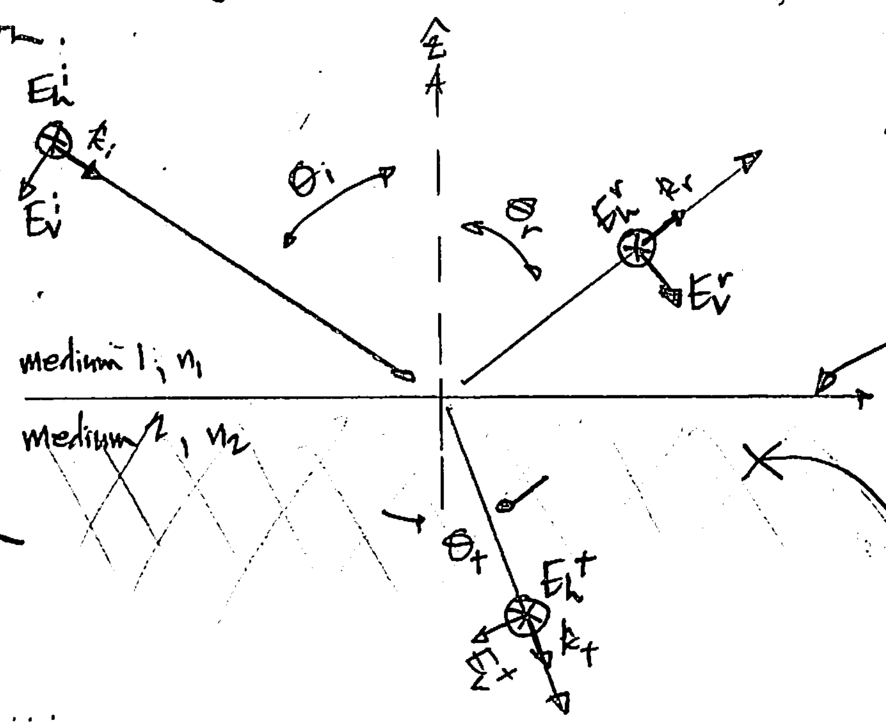
Electromagnetic Radiation at a Boundary

When electromagnetic radiation encounters the boundary between two media with distinct electrical properties, some of the radiation is scattered: some radiation propagates back into the first medium (reflection); some radiation propagates into the second medium (transmission).

In microwave remote sensing, we consider horizontally- and vertically-polarized radiation.

$\hat{h}$ = horizontal pol unit vector $\equiv \frac{\hat{z}\times\hat{k}}{|\hat{z}\times\hat{k}|}$

$\hat{k}$ = direction of propagation



$\hat{v}$ = vertical pol unit vector $\equiv \hat{h}\times\hat{k}$

specular (flat) surface

plane of incidence is the paper

where ...

Γ_h = h-pol Fresnel reflection coefficient for electric field $= \Gamma_\perp$

$\Gamma_v = -\Gamma_\parallel$ due to way fields were defined!

$T_h = T_\perp$

$T_v = T_\parallel$

$T_h = 1 + \Gamma_h$, $\quad T_v = (1-\Gamma_v)\frac{\cos\theta_i}{\cos\theta_t}$

physical explanation using pattern for a dipole

θ_B = Brewster angle = incidence angle at which $\Gamma_v = 0$ (no Brewster angle for h-pol!)

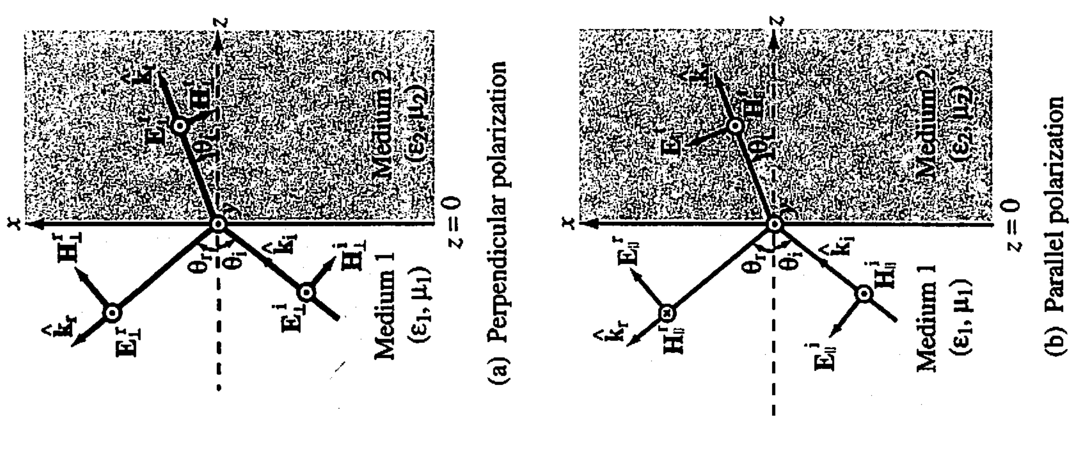


Figure 8-14: The plane of incidence is the plane containing the direction of wave travel, $\hat{\mathbf{k}}$, and the surface normal to the boundary, which in the present case is the plane of the paper. A wave is (a) perpendicularly polarized when its **E** is perpendicular to the plane of incidence and (b) parallel polarized when its **E** lies in the plane of incidence.

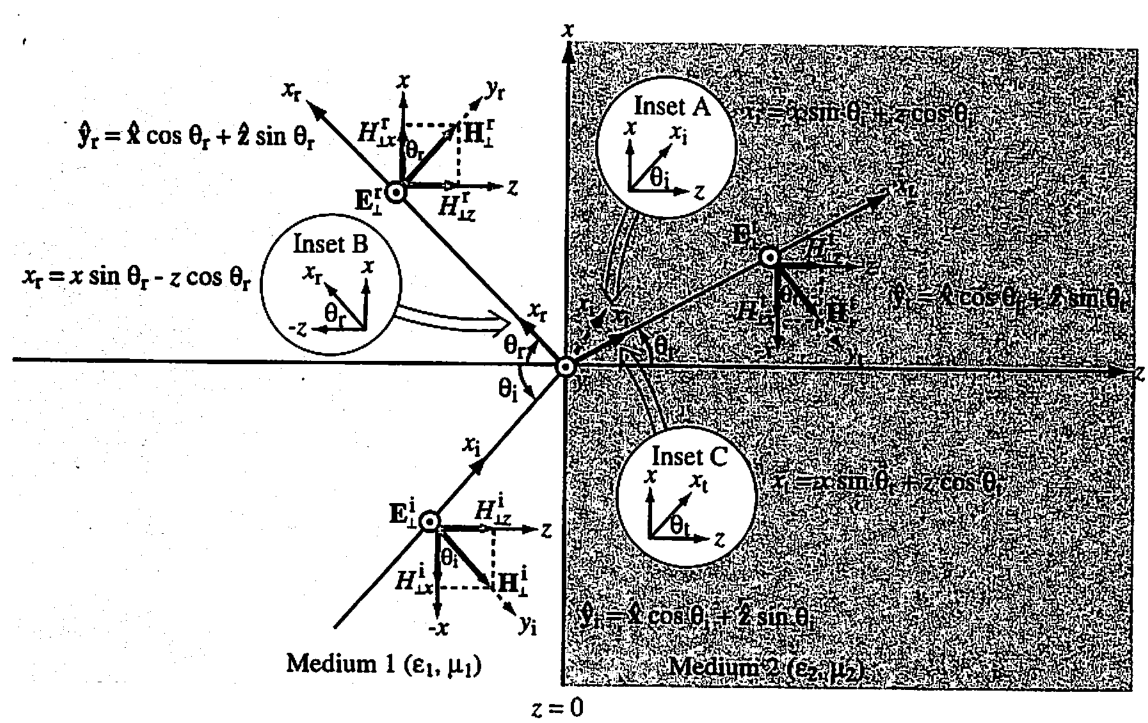


Figure 8-15: Perpendicularly polarized plane wave incident at an angle θ_i upon a planar boundary.

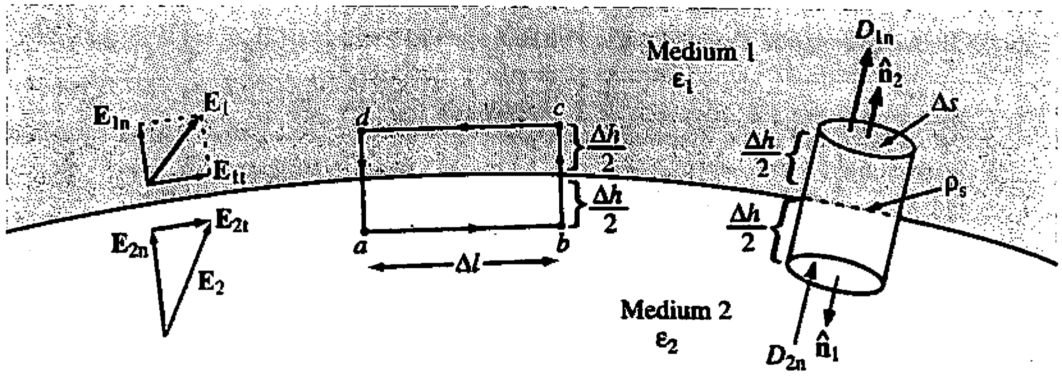


Figure 4-18: Interface between two dielectric media.

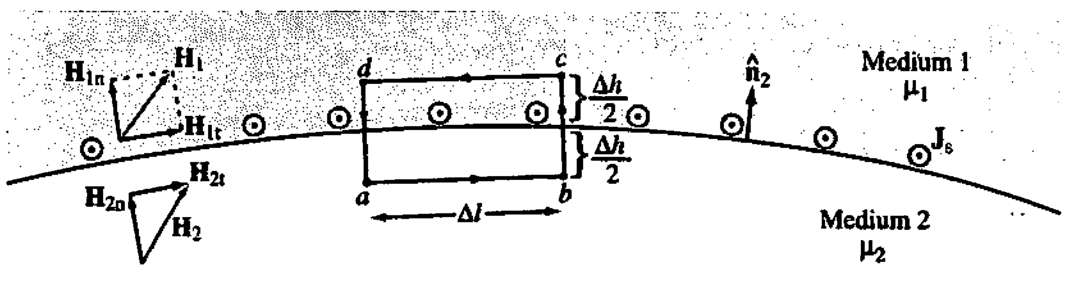


Figure 5-24: Boundary between medium 1 with μ_1 and medium 2 with μ_2.

Table 6-2: Boundary conditions for the electric and magnetic fields.

Field Components	General Form	Medium 1 Dielectric	Medium 2 Dielectric	Medium 1 Dielectric	Medium 2 Conductor
Tangential E	$\hat{\mathbf{n}}_2 \times (\mathbf{E}_1 - \mathbf{E}_2) = 0$	$E_{1t} = E_{2t}$		$E_{1t} = E_{2t} = 0$	
Normal D	$\hat{\mathbf{n}}_2 \cdot (\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$	$D_{1n} - D_{2n} = \rho_s$		$D_{1n} = \rho_s$	$D_{2n} = 0$
Tangential H	$\hat{\mathbf{n}}_2 \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$	$H_{1t} = H_{2t}$		$H_{1t} = J_s$	$H_{2t} = 0$
Normal B	$\hat{\mathbf{n}}_2 \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$	$B_{1n} = B_{2n}$		$B_{1n} = B_{2n} = 0$	

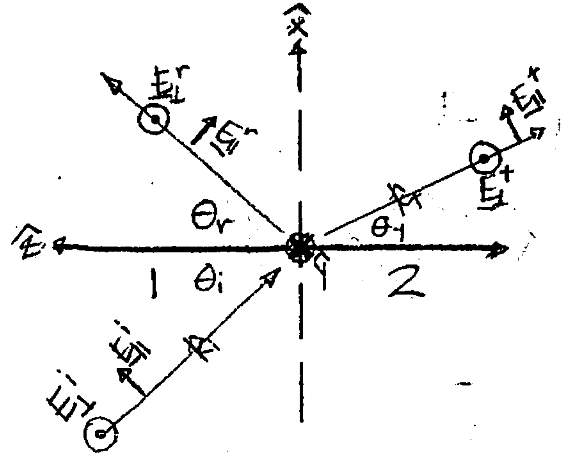
Notes: (1) ρ_s is the surface charge density at the boundary; (2) $\mathbf{J}_s$ is the surface current density at the boundary; (3) normal components of all fields are along $\hat{\mathbf{n}}_2$, the outward unit vector of medium 2; (4) $E_{1t} = E_{2t}$ implies that the tangential components are equal in magnitude and parallel in direction; (5) direction of $\mathbf{J}_s$ is orthogonal to $(\mathbf{H}_1 - \mathbf{H}_2)$.

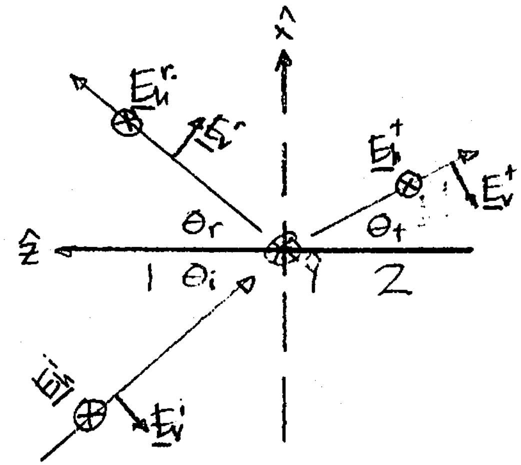
$$\hat{h} \triangleq \frac{\hat{z}\times\hat{k}}{|\hat{z}\times\hat{k}|} \qquad \hat{v} \triangleq \hat{h}\times\hat{k}$$

$\hat{k}^i_h = -\hat{z} + \hat{x}$

$\hat{z}\times(-\hat{z}+\hat{x}) = \hat{z}\times\hat{x} = \hat{y}$

$\hat{k}^r_h = \hat{z}+\hat{x}$





$$\Gamma_\perp = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} \qquad \Gamma_h = \Gamma_\perp = \frac{n_1\cos\theta_i - n_2\cos\theta_t}{n_1\cos\theta_i + n_2\cos\theta_t} \triangleq \frac{E_h^r}{E_h^i}$$

$$T_\perp = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} \qquad T_h = T_\perp = \frac{2n_1\cos\theta_i}{n_1\cos\theta_i + n_2\cos\theta_t} \triangleq \frac{E_h^t}{E_h^i}$$

$$\Gamma_\parallel = \frac{n_1\cos\theta_t - n_2\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i} \qquad \Gamma_v = -\Gamma_\parallel = \frac{n_2\cos\theta_i - n_1\cos\theta_t}{n_2\cos\theta_i + n_1\cos\theta_t} \triangleq \frac{E_v^r}{E_v^i}$$

$$T_\parallel = \frac{2n_1\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i} \qquad T_v = T_\parallel = \frac{2n_1\cos\theta_i}{n_1\cos\theta_t + n_2\cos\theta_i} \triangleq \frac{E_v^t}{E_v^i}$$

$$T_\perp = 1 + \Gamma_\perp, \quad T_\parallel = (1+\Gamma_\parallel)\frac{\cos\theta_i}{\cos\theta_t} \qquad T_h = 1+\Gamma_h, \quad T_v = (1-\Gamma_v)\frac{\cos\theta_i}{\cos\theta_t}$$

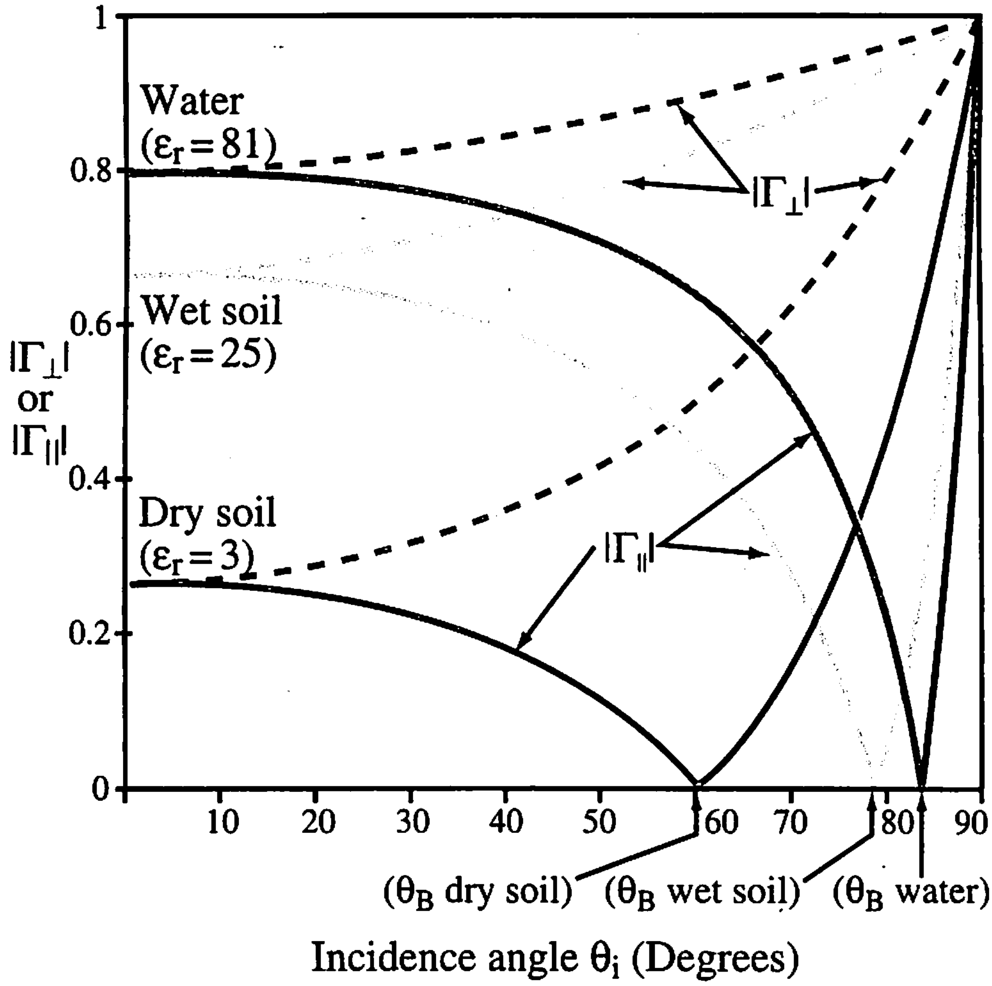


Figure 8-17: Plots for $|\Gamma_\perp|$ and $|\Gamma_\parallel|$ as a function of θ_i for a dry soil surface, a wet-soil surface, and a water surface. For each surface, $|\Gamma_\parallel| = 0$ at the Brewster angle.