Microwave Radiometry & Earth's Surface

Recall 3 case examples:

1. Microwave radiometry of Earth atmosphere \( \rightarrow \) total precipitable water (emission only)

2. Microwave radiometry of Earth atmosphere \( \rightarrow \) clouds and precipitation (rain and snow) (TRMM)

3. Microwave radiometry of Earth surface \( \rightarrow \) soil moisture, sea surface salinity, vegetation biomass (SMOS, the European Space Agency's Soil Moisture and Ocean Salinity mission to be launched later this year)

- Satellite, airplane, tower

At low microwave frequencies both for surface remote sensing atmospheric emission and atmospheric emission reflected by the surface are both small and can be neglected.

\( T_b \) 

\( T_{b,\text{surface}} \) 

- Smooth or rough bare soil surface or ocean surface
Reflection and Transmission

What happens to an electromagnetic wave when it encounters the boundary between two media with different electrical properties?

Must first make definitions

parallel polarization: wave whose electric field is parallel to the plane of incidence

perpendicular polarization: ... electric field perpendicular ...

plane of incidence: plane containing the normal to the boundary and the direction of propagation of incident wave

plane of incidence is the x-z plane

perpendicular polarization

see Figure 8-14

in handout

\[ \hat{A} = \frac{1}{2} \mathbf{k} \times \mathbf{E} \]

\[ \mathbf{E} = -\mathbf{r} \times \hat{A} \]

\( \mathbf{r} = \) direction of propagation
Examine perpendicular polarization.

To find the relationship between incident, reflected, and transmitted fields, must use boundary conditions.

\[
\begin{align*}
\text{incident wave:} & \quad \mathbf{E}_i = \hat{y} \mathbf{E}_0 \ e^{-j \mathbf{k}_i \cdot (x \sin \theta_i + z \cos \theta_i)} \\
\mathbf{H}_i &= (-\hat{x} \cos \theta_i + \hat{z} \sin \theta_i) \ \mathbf{E}_0 \ e^{-j \mathbf{k}_i \cdot (x \sin \theta_i + z \cos \theta_i)} \\
\text{reflected wave:} & \quad \mathbf{E}_r = \hat{y} \mathbf{E}_0 \ e^{j \mathbf{k}_i \cdot (x \sin \theta_i - z \cos \theta_i)} \\
\mathbf{H}_r &= (\hat{x} \cos \theta_i + \hat{z} \sin \theta_i) \ \mathbf{E}_0 \ e^{j \mathbf{k}_i \cdot (x \sin \theta_i - z \cos \theta_i)} \\
\text{transmitted wave:} & \quad \mathbf{E}_t = \ldots \\
\mathbf{H}_t &= \ldots
\end{align*}
\]

BC's:

\[
k_1 \times \sin \theta_i = k_1 \times \sin \theta_r = k_2 \times \sin \theta_t
\]

"phase matching condition"

Reference: Chapters 8 & 9 of Uday's Fundamentals of Applied Electromagnetics, Section 2-6 & UMF

Boundary conditions:

1. Total field in medium 1:
   \[
   \mathbf{E}' = \mathbf{E}_i + \mathbf{E}_r \\
   \mathbf{H}' = \mathbf{H}_i + \mathbf{H}_r
   \]
2. Total field in medium 2:
   \[
   \mathbf{E}_2 = \mathbf{E}_t \\
   \mathbf{H}_2 = \mathbf{H}_t
   \]

Apply boundary conditions: tangential fields must be equal!
phase matching condition

\[ k_1 \times \sin \theta_i = k_1 \times \sin \theta_r = k_2 \times \sin \theta_t \]

\[ \frac{k_1}{n_1} \sin \theta_i = \frac{k_2}{n_2} \sin \theta_t \]

Snell's law of refraction

\[ n_1 \sin \theta_i = n_2 \sin \theta_t \]

\[ n = \frac{\sqrt{\mu}}{c} \]

\[ \mu = \mu_i = \frac{2 \pi n}{\lambda_0} \]

\[ \frac{2 \pi n}{\lambda_0} = \frac{2 \pi \mu_i}{c} \]

\[ \lambda_0 = \frac{c}{\sqrt{\mu}} \]

\[ \mu = \frac{\sqrt{\mu \mu_0}}{c} \]

\[ \lambda_0 = \frac{c}{\sqrt{\mu \mu_0}} \]

\[ \lambda_0 = \frac{c}{\sqrt{\mu \mu_0}} \]

More geometry...

\[ \Gamma_\perp = \frac{E_\perp^r}{E_\perp^i} = \text{Fresnel reflection coefficient for perpendicular polarization} = \frac{E_\perp^r}{E_\perp^i} = \frac{n_2 \cos \theta_i - n_1 \cos \theta_r}{n_1 \cos \theta_i + n_2 \cos \theta_r} \]

\[ \mu_{\perp} = \frac{2 n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_r} \]

\[ T_\perp = \frac{E_\perp^t}{E_\perp^i} = \text{Fresnel transmission coefficient for perpendicular polarization} = \frac{2 n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_r} \]

\[ T_\perp = 1 + \Gamma_\perp \]

Note these Fresnel coefficients are for perpendicular polarization of electric fields - different expressions for magnetic fields and for parallel polarization.
Electromagnetic Radiation at a Boundary

When electromagnetic radiation encounters the boundary between two media with distinct electrical properties, some of the radiation is scattered: some radiation propagates back into the first medium (reflection), some radiation propagates into the second medium (transmission).

In microwave remote sensing, we consider horizontally- and vertically-polarized radiation.

\[ \hat{E}_h = \text{horizontal pol. unit vector} \]
\[ \hat{E}_v = \text{vertical pol. unit vector} \]
\[ \hat{k} = \text{direction of propagation} \]

\[ \Gamma_h = \text{h-pol Fresnel reflection coefficient} = \Gamma_\perp \text{ for electric field} \]
\[ \Gamma_v = -\Gamma_\parallel \text{ due to way fields were defined!} \]
\[ T_h = T_\perp \]
\[ T_v = T_\parallel \]
\[ T_h = 1 + \Gamma_h \]
\[ T_v = (1 - \Gamma_v) \frac{\cos \Theta_i}{\cos \Theta_f} \]

\[ \Theta_i = \text{Incidence angle} \]
\[ \Gamma_v = 0 \text{ (no Brewster angle for h-pol!) \} }
Figure 8.14: The plane of incidence is the plane containing the direction of wave travel, k, and the surface normal to the boundary, which in the present case is the plane of the paper. A wave is (a) perpendicular to the plane of incidence and (b) parallel polarized when its E is perpendicular to the plane of incidence.

Figure 8.15: Perpendicularly polarized plane wave incident at an angle $\theta_i$ upon a planar boundary.
Table 6-2: Boundary conditions for the electric and magnetic fields.

<table>
<thead>
<tr>
<th>Field Components</th>
<th>General Form</th>
<th>Medium 1 Dielectric</th>
<th>Medium 2 Dielectric</th>
<th>Medium 1 Dielectric</th>
<th>Medium 2 Conductor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tangential E</td>
<td>$\hat{n}_2 \times (E_1 - E_2) = 0$</td>
<td>$E_{11} = E_{21}$</td>
<td></td>
<td></td>
<td>$E_{11} = E_{21} = 0$</td>
</tr>
<tr>
<td>Normal D</td>
<td>$\hat{n}_2 \cdot (D_1 - D_2) = \rho_s$</td>
<td>$D_{1n} = D_{2n} = \rho_s$</td>
<td></td>
<td></td>
<td>$D_{1n} = \rho_s$</td>
</tr>
<tr>
<td>Tangential H</td>
<td>$\hat{n}_2 \times (H_1 - H_2) = J_s$</td>
<td>$H_{11} = H_{21}$</td>
<td></td>
<td></td>
<td>$H_{11} = J_s$</td>
</tr>
<tr>
<td>Normal B</td>
<td>$\hat{n}_2 \cdot (B_1 - B_2) = 0$</td>
<td>$B_{1n} = B_{2n}$</td>
<td></td>
<td></td>
<td>$B_{1n} = B_{2n} = 0$</td>
</tr>
</tbody>
</table>

Notes: (1) $\rho_s$ is the surface charge density at the boundary; (2) $J_s$ is the surface current density at the boundary; (3) normal components of all fields are along $\hat{n}_2$, the outward unit vector of medium 2; (4) $E_{11} = E_{21}$ implies that the tangential components are equal in magnitude and parallel in direction; (5) direction of $J_s$ is orthogonal to $(H_1 - H_2)$. 

Figure 4-18: Interface between two dielectric media.

Figure 5-24: Boundary between medium 1 with $\mu_1$ and medium 2 with $\mu_2$. 


\[ \Gamma_\perp = \frac{n_1 \cos \Theta_i - n_2 \cos \Theta_t}{n_1 \cos \Theta_i + n_2 \cos \Theta_t} \]

\[ \Gamma_{\parallel} = \frac{Z n_1 \cos \Theta_i}{n_1 \cos \Theta_i + n_2 \cos \Theta_i} \]

\[ \Gamma_{\perp} = \frac{2 n_1 \cos \Theta_i}{n_1 \cos \Theta_i + n_2 \cos \Theta_i} \]

\[ \Gamma_{\parallel} = \frac{n_1 \cos \Theta_t - n_2 \cos \Theta_i}{n_1 \cos \Theta_t + n_2 \cos \Theta_t} \]

\[ \Gamma_{\parallel} = \frac{Z n_1 \cos \Theta_t}{n_1 \cos \Theta_i + n_2 \cos \Theta_i} \]

\[ \Gamma_{\perp} = \frac{2 n_1 \cos \Theta_t}{n_1 \cos \Theta_i + n_2 \cos \Theta_i} \]

\[ T_{\perp} = 1 + \Gamma_{\perp}, \quad T_{\parallel} = (1 + \Gamma_{\parallel}) \frac{\cos \Theta_i}{\cos \Theta_t} \]

\[ T_{\perp} = \frac{2 n_1 \cos \Theta_i}{n_1 \cos \Theta_i + n_2 \cos \Theta_i} \]

\[ T_{\parallel} = \frac{Z n_1 \cos \Theta_i}{n_1 \cos \Theta_i + n_2 \cos \Theta_i} \]

\[ T_{\perp} = \frac{2 n_1 \cos \Theta_t}{n_1 \cos \Theta_i + n_2 \cos \Theta_i} \]

\[ T_{\parallel} = \frac{Z n_1 \cos \Theta_t}{n_1 \cos \Theta_i + n_2 \cos \Theta_i} \]

\[ T_{\perp} = \frac{2 n_1 \cos \Theta_t}{n_1 \cos \Theta_i + n_2 \cos \Theta_i} \]

\[ T_{\parallel} = \frac{Z n_1 \cos \Theta_t}{n_1 \cos \Theta_i + n_2 \cos \Theta_i} \]
Figure 8-17: Plots for $|\Gamma_\perp|$ and $|\Gamma_\parallel|$ as a function of $\theta_i$ for a dry soil surface, a wet-soil surface, and a water surface. For each surface, $|\Gamma_\parallel| = 0$ at the Brewster angle.