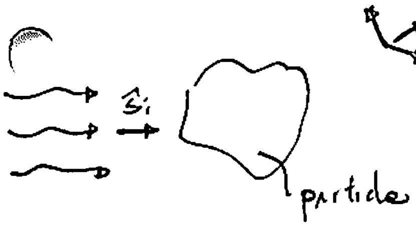
4/15/08

# Radiative Transfer and Scattering




$\sigma^{pq}_{scat}(\hat{s}_i)$ = scattering cross section

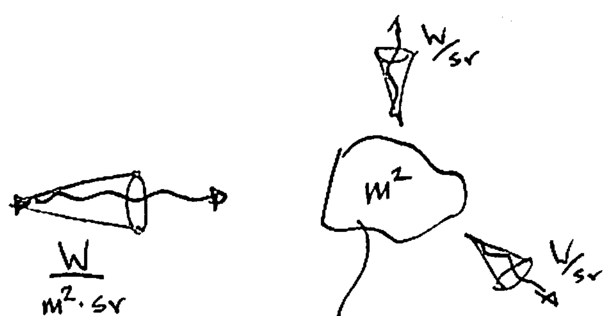
$$= \frac{\text{q-polarized scattered power}}{\text{q-polarized incident power density}}$$

$$= \int_{4\pi} |S_{pq}(\hat{s}_s, \hat{s}_i)|^2 \, d\Omega_s$$ integrate over all scattered directions

where $E_{p_{scat}}(\hat{s}_s) = \frac{e^{-jkR}}{R} S_{pq}(\hat{s}_s, \hat{s}_i) E_{q_{inc}}(\hat{s}_i)$

$$\sigma_{scat}(\hat{s}_i) = \sigma^{pp}_{scat}(\hat{s}_i) + \sigma^{pq}_{scat}(\hat{s}_i) + \sigma^{qq}_{scat}(\hat{s}_i) + \sigma^{qp}_{scat}(\hat{s}_i) = \frac{\text{total scattered power}}{\text{total incident power density}}$$

$$[\sigma_{scat}(\hat{s}_i)] = m^2$$




power that is scattered out of original beam

effective scattering area <u>intercepts</u> incident brightness, particle scatters (re-radiates) in all directions

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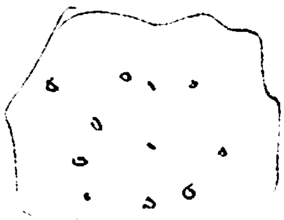
# Multiple particle scattering



$N$ = number of identical particles per volume

$\kappa_s(\hat{s})$ = scattering coefficient $= N\sigma_{scat}(\hat{s})$ — remember assumptions

$[\kappa_s] = \frac{\#}{m^3} \times m^2 = Np \cdot m^{-1}$

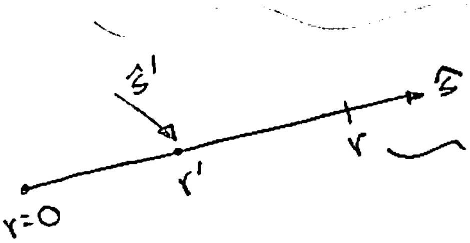
Similarly, $\kappa_{a_p}(\hat{s})$ = absorption coefficient for particles $= N\sigma_{abs}(\hat{s})$

$$\sigma_{abs}(\hat{s}_i) = \frac{\text{total power absorbed by single particle}}{\text{total power density incident from } \hat{s}_i}$$

$$P(\hat{s},\hat{s}') = \text{scattering phase function} = N\left\{|S_{pq}(\hat{s},\hat{s}')|^2 + |S_{pp}(\hat{s},\hat{s}')|^2 + \ldots\right\}$$

$$= \frac{\kappa_s}{4\pi}\,\psi(\hat{s},\hat{s}') \quad , \quad \psi(\hat{s},\hat{s}') = \text{normalized phase function}$$

## Summary




$T_B(r,\hat{s})$ = unpolarized brightness temperature at $r$ in the $\hat{s}$ direction

$$T_B(r,\hat{s}) = T_B(0,\hat{s})\,e^{-\tau(0,r)} + \int_0^r \left\{\kappa_a(r')T(r') + \frac{\kappa_s}{4\pi}\int_{4\pi}\psi(\hat{s},\hat{s}')\,T_B(r',\hat{s}')\,d\Omega'\right\} e^{-\tau(r',r)}\,dr'$$

$$\tau(r_1,r_2) = \int_{r_1}^{r_2}\kappa_e(r)\,dr$$

$\kappa_e = \kappa_a + \kappa_s$

$\kappa_a = \kappa_{ab} + \kappa_{a_p}$

$\kappa_{ab} = 2k_0 n''$ <u>or</u> expressions for water vapor and/or oxygen

$\kappa_{ab} = N\sigma_{abs}$

$\kappa_s = N\sigma_{scat}$

## General Solution of Radiative Transfer

correct

$$T_b(r,\hat{s}) = T_b(0,\hat{s})\, e^{-\tau(0,r)} + \int_0^r \kappa_e(r')\left[(1-a)\,T(r') + a\, T_{sc}(r',\hat{s})\right] e^{-\tau(r',r)}\, dr'$$

$$T_b(r,\hat{s}) = \text{brightness temperature in the } \hat{s} \text{ direction, K}$$

$$B(r,\hat{s}) = \frac{2\mathrm{k}}{\lambda^2}\, T_b(r,\hat{s}) = \text{spectral brightness, } \mathrm{W \cdot m^{-2} \cdot sr^{-1} \cdot Hz^{-1}}$$

$$\mathrm{k} = 1.3807 \times 10^{-23} = \text{Boltzmann's constant, } \mathrm{J \cdot K^{-1}}$$

$$\tau(r_1, r_2) \equiv \int_{r_1}^{r_2} \kappa_e(r')\, dr' = \text{optical depth from } r_1 \text{ to } r_2$$

$$T(r) = \text{physical temperature of the medium}$$

$$T_{sc}(r,\hat{s}) = \tfrac{1}{4\pi} \int_{4\pi} \psi(\hat{s},\hat{s}')\, T_b(r,\hat{s}')\, d\Omega' \;=\; \tfrac{1}{2} \int_{-1}^{1} \psi(\mu,\mu')\, T_b(r,\mu')\, d\mu'$$

Rewriting:

$$T_b(r,\hat{s}) = T_b(0,\hat{s})\, e^{-\tau(0,r)} + \int_0^r \left\{ \kappa_{bg}(r')\, T(r') + \int_{4\pi} P(\hat{s},\hat{s}')\, T_b(r',\hat{s}')\, d\Omega' \right\} e^{-\tau(r',r)}\, dr'$$

## Coefficients

$$\kappa_e = \kappa_{bg} + \kappa_{part} = \text{extinction coefficient, } \mathrm{Np \cdot m^{-1}}$$

$$\kappa_{bg} = \text{background absorption coefficient, } \mathrm{Np \cdot m^{-1}}$$

$$\kappa_{part} = N\,\sigma_{part} = \text{particle extinction coefficient, } \mathrm{Np \cdot m^{-1}}$$

$$N = \text{number of identical particles per cubic meter, } \mathrm{m^{-3}}$$

$$\sigma_{part} = \sigma_a + \sigma_s = \text{extinction cross section of a single particle, } \mathrm{m^2}$$

$$\sigma_a = \text{particle absorption cross section, } \mathrm{m^2}$$

$$\sigma_s = \text{particle scattering cross section, } \mathrm{m^2}$$

$$a = \frac{\kappa_{part}}{\kappa_e} = \text{albedo}$$

$N\sigma_{prt}$ or $N\sigma_s$

## Particle Interaction

$$P(\hat{s},\hat{s}') = N\left|S_{pp}(\hat{s},\hat{s}')\right|^2 = \frac{\kappa_{part}}{4\pi}\psi(\hat{s},\hat{s}') = \text{phase function, units m}^{-1}$$

$$\psi(\hat{s},\hat{s}') = \text{normalized phase function}$$

$$E_p^{scattered}(\hat{s}) = \frac{e^{-jkr}}{r} S_{pp}(\hat{s},\hat{s}')\, E_p^{incident}(\hat{s}')$$

$$\sigma_s^{pp}(\hat{s}') = \int_{4\pi}\left|S_{pp}(\hat{s},\hat{s}')\right|^2 d\Omega_s = \frac{\text{power scattered (all directions } \hat{s})}{\text{incident power density } (\hat{s}')}$$

$$\sigma_a = \frac{-\int_S \mathrm{Re}\left\{\frac{1}{2}\mathbf{E}\times\mathbf{H}^*\right\}\cdot\hat{n}\, da}{\frac{1}{2\eta_o}\left|E_p^{inc}(\hat{s})\right|^2} = \frac{\text{net power flow into particle}}{\text{incident power density}}$$

$$\sigma_a = \frac{\int_V \frac{1}{2}\omega\varepsilon''\left|\mathbf{E}(\mathbf{r}')\right|^2 dv'}{\frac{1}{2\eta_o}\left|E_p^{inc}(\hat{s})\right|^2} = \frac{\text{dissipated power in particle volume}}{\text{incident power density}}$$

## Media Constants

$$\kappa_{bg} = \text{background absorption coefficient} = 2\alpha$$

$$\alpha = \text{attenuation constant} = \omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right\}^{\frac{1}{2}} = k_o\left|\mathrm{Im}\left\{\sqrt{\varepsilon_r}\right\}\right|_{\mu=\mu_o}$$

$$k = \text{wave number (phase constant)} = \omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right\}^{\frac{1}{2}} = k_o\left|\mathrm{Re}\left\{\sqrt{\varepsilon_r}\right\}\right|_{\mu=\mu_o}$$

exact

approximation