Thermal Emission

The Planck law describes the electromagnetic radiation that would be naturally emitted by a blackbody (a perfect emitter), if one existed. All matter emits radiation as a result of the motions of the molecules which compose all matter. As these molecules transition among energy states, radiation is emitted. Since temperature is a measure of the average kinetic energy of molecules, and since these transitions increase as the kinetic energy of the molecules increase, the emission of radiation is caused by and increases with the temperature of a material. This is thermal emission. Note that there are other processes that cause matter to emit radiation (P6-3).

What is the frequency of maximum radiation?

\[ \frac{dE}{d\nu} = 0 \quad \frac{d^2E}{d\nu^2} = 0 \quad \nu_{\text{max}} = 5.87 \times 10^{10} \text{T} \]

\([\nu_{\text{max}}] = \text{Hz}, \ [\text{T}] = \text{K}\)

What is the wavelength of maximum radiation?

\[ \frac{dE}{d\lambda} = 0 \quad \lambda_{\text{max}} T = 2.897 \times 10^{-3} \]

\([\lambda_{\text{max}}] = \text{m}, \ [\text{T}] = \text{K}\)

This is called the Wien displacement law?
Planck law approximations

\[ \text{When } \frac{hc}{\lambda K T} \gg 1, \quad \frac{1}{e^{\frac{hc}{\lambda K T}} - 1} = \frac{1}{e^{\frac{hc}{\lambda K T}}} = e^{\frac{-hc}{\lambda K T}} \]

and then

\[ B_\lambda (\lambda, T) = \frac{2hc^2}{\lambda^5} e^{\frac{-hc}{\lambda K T}} \]

\[ \text{Rayleigh–Jeans law}\]

valid for short wavelengths
at typical Earth temperatures

\[ \text{When } \frac{hf}{kT} \ll 1, \quad e^{\frac{hf}{kT}} - 1 = \frac{hf}{kT} \]

and then

\[ B_\nu (\nu, T) = \frac{2k}{\nu^2} T \]

Rayleigh–Jeans law
valid for low frequencies
at typical Earth temperatures

What is so great about spectral brightness?

\[ B_\lambda (\lambda, T, \Theta, \phi), \quad \text{W} \cdot \text{m}^{-2} \cdot \text{sr}^{-1} \cdot \text{Hz}^{-1} \]

Area emitting

see next page

An important property of spectral brightness is that it is constant along any ray in free space! Spectral brightness is invariant with distance, it doesn't "spread out" like power density (recall

\[ S_R (R, \Theta, \phi) = \frac{1}{R^2} \]). Any change in spectral brightness is due to the properties of the medium through which the radiation propagates! The same thing goes for brightness (W \cdot m^{-2} \cdot sr^{-1}).
\[ \hat{n} = \text{normal to } ds \]

\[ d\sigma' \text{ at point } Q' \]

\[ d\sigma = \text{differential area} \]

\[ B_s(Q) = \text{spectral brightness at point } Q \]

\[ [B_s(Q)] = \text{W mm}^{-2} \text{ sr}^{-1} \text{ Hz}^{-1} \]

\[ \text{energy incident on } d\sigma \text{ at point } Q \]

\[ dE = (B_s(Q) \hat{\mathbf{r}} \cdot d\hat{\mathbf{e}}) d\Omega \, df \, dt, \quad [dE] = J \]

\[ = B_s(Q) \hat{\mathbf{r}} \cdot \mathbf{R} \, d\sigma \, d\Omega \, df \, dt \]

\[ = B_s(Q) \cos\Theta \, d\sigma \, \frac{\cos\Theta \, d\sigma'}{R^2} \, df \, dt = \frac{B_s(Q) \cos\Theta \cos\Theta' \, d\sigma \, d\sigma'}{R^2} \, df \, dt \]

\[ \text{energy emitted by } d\sigma' \text{ at } Q' \]

\[ dE' = B_s(Q') \hat{\mathbf{r}}' \cdot d\hat{\mathbf{e}}' \, d\Omega' \, df \, dt \]

\[ = B_s(Q') \hat{\mathbf{r}}' \cdot \mathbf{R} \, d\sigma' \, d\Omega' \, df \, dt \]

\[ = B_s(Q') \cos\Theta' \, d\sigma' \, \frac{\cos\Theta \, d\sigma}{R^2} \, df \, dt \]

\[ dE' = \frac{B_s(Q') \cos\Theta' \cos\Theta \, d\sigma \, d\sigma'}{R^2} \, df \, dt \]

\[ B_s(Q) = B_s(Q') ! ! ! \text{ (spectral brightness invariant with fixture)} \]
Total Hemispherical Radiation Emitted by a Blackbody Surface

$$\hat{n} = \hat{R} \cos \theta - \hat{S} \sin \theta$$

$$R_{\hat{n}}(\theta, \phi) \ dA$$

$$P_{\text{rad}} = \text{power radiated} = \int \int \int_{A} B_{\hat{n}}(\theta, \phi) \cdot dA = 4\pi A$$

$$B = \int_{0}^{\phi} B_{\hat{n}}(\phi, T) \ d\phi = \text{total brightness over all frequencies} = \frac{\sigma T^4}{\pi}, \ W \cdot m^{-2} \cdot sr^{-1}$$

**Stefan-Boltzmann law**

$$\sigma = 5.67 \times 10^{-8} \ W \cdot m^{-2} \cdot K^{-4} \cdot sr^{-1}$$

$$P_{\text{rad}} = \int \int B(\theta, \phi) \hat{n} \cdot dA \ dS = \int \int B(\theta, \phi) \hat{n} \cdot (\hat{R} \cos \theta - \hat{S} \sin \theta) \ dA \ dS$$

$$= \int \int B(\theta, \phi) \cos \theta \ d\Omega \ da$$

**cosine law:** area radiating decreases with \(\theta\)

Blackbody brightness is isotropic

$$P_{\text{rad}} = \int \int B(\phi, T) \cos \theta \ d\Omega \ da$$

the same in every direction!

More familiar:

$$\sigma T^4 = \text{radiant emittance} \ W \cdot m^{-2}$$

**Stefan-Boltzmann law**

$$\sigma T^4 = \text{power radiated by surface area} A, W$$