Antennas and Radiation

Recall: \( S_{av} = \text{time-average propagating power} = \frac{1}{2} \text{Re} \{ E \times H^* \} = \frac{|E|^2}{2\eta_0} \)

In the "far field" of an antenna, \((R > \frac{2D^2}{\lambda}, D = \text{largest dimension} \& \text{antenna aperture})\),

\[
E = \hat{E}E_0 + \phi E_0, \quad E_0 = e^{-i\frac{kR}{\rho}} \frac{1}{R} f_0(\theta, \phi), \quad E_\phi = e^{-i\frac{kR}{\rho}} \frac{R}{\rho} f_\phi(\theta, \phi)
\]

Spherical propagation function

directional characteristics of antenna

\[
S_{av} = \hat{R} \frac{|E|^2}{2\eta_0} = \hat{R} \frac{|E_0|^2 + |E_\phi|^2}{2\eta_0} = \hat{R} S_R(R, \theta, \phi)
\]

\( S_R(R, \theta, \phi) = \hat{R} \text{ component of time-average propagating vector} \)

Power density of propagating wave \( [S_R] = \frac{W}{m^2} \)

Sun radiates isotropically

If \( S_R(R, \theta, \phi) = S_R(R) \) = \( \frac{1}{4\pi} \frac{W}{m^2} \)

then how much power is radiated by the Sun?

\[
Prad = S_R(R_e) \cdot (\text{area of sphere} R_e) = \left( 10^3 \frac{W}{m^2} \right) \left( \frac{4}{3} \pi (1.5 \times 10^{11} m)^2 \right) = 2.8 \times 10^{26} W
\]

What is \( S_R(R) \)?

\[
S_R(R) = \frac{Prad}{4\pi R^2} = \frac{Prad}{4\pi R_e^2} = 2.8 \times 10^{26} W > S_R(R_e)
\]

by conservation of energy!

What is \( S_R(R_m) \)?

\[
S_R(R_m) = \frac{Prad}{4\pi R_m^2} < S_R(R_e)
\]

In general: \( S_R(R) = \frac{Prad}{4\pi R^2} \quad \Rightarrow \quad S_R(R, \theta, \phi) = \frac{1}{R^2} \)
Antenna Pattern Dimensions

\[ P_{\text{rad}} = \int F(\theta, \phi) \, d\Omega = \int F_{\text{max}} F_n(\theta, \phi) \, d\Omega = F_{\text{max}} \Omega_p \]

where \( \Omega_p = \int F_n(\theta, \phi) \, d\Omega = \text{pattern solid angle} \), \( [\Omega_p] = \text{sr} \)

= the solid angle through which all the power radiated by the antenna would flow if its radiation pattern were constant and equal to \( F_{\text{max}} \) for all angles inside \( \Omega_p \).

see Fig 9-11

\[ P_{\text{rad}} = F_{\text{max}} \Omega_p \]

\[ \Omega_p = \frac{P_{\text{rad}}}{F_{\text{max}}} \]

\[ S = \frac{P_{\text{rad}}}{P_{\text{total}}} \]

\[ \Omega_p = 10 \log(50\%) \]

\[ B_{x,y} = 3 \text{dB beamwidth in } x-y \text{ plane} = 2 \Theta_{x,y} \]

where \( F_n(\Theta_{x,y}, \phi = 0) = 0.5 = -3 \text{dB} \) See Fig 9-12

\[ B_{y,z} = 3 \text{dB beamwidth in } y-z \text{ plane} = 2 \Theta_{y,z} \]

where \( F_n(\Theta_{y,z}, \phi = \frac{\pi}{2}) = 0.5 = -3 \text{dB} \)

\[ \Omega_p = B_{x,y} B_{y,z} \]

\[ \Omega_M = \text{solid angle of main lobe} = \int F_n(\theta, \phi) \, d\Omega \]

\[ M = \frac{\Omega_M}{\Omega_p} = \text{main beam efficiency} = \text{the percentage of the pattern solid angle in the main beam} \]
Directivity and Gain

The directivity of an antenna is an important "figure of merit" or characteristic that gives the direction properties of the antenna as compared to an isotropic antenna.

\[ D(\Theta, \Phi) = \text{directivity} = \frac{\int \int \int R(\Theta, \Phi, \varsigma) \, d\varsigma}{\int \int \int R(\Theta, \Phi, \varsigma) \, d\varsigma} = \frac{\text{power density in direction } (\Theta, \Phi)}{\text{power density if equivalent isotropic antenna}} \]

\[ D(\Theta, \Phi) = \frac{\int \int \int R(\Theta, \Phi, \varsigma) \, d\varsigma}{\int \int \int R(\Theta, \Phi, \varsigma) \, d\varsigma} = \frac{\int \int \int R(\Theta, \Phi, \varsigma) \, d\varsigma}{\int \int \int R(\Theta, \Phi, \varsigma) \, d\varsigma} = \frac{\text{radiation intensity in direction } (\Theta, \Phi)}{\text{radiation intensity if equivalent isotropic antenna}} \]

\[ D(\Theta, \Phi) = \frac{\text{radiation intensity in direction } (\Theta, \Phi)}{\text{average radiation intensity}} \]

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Gain is just like directivity but more realistic as it takes into account losses (energy sent to antenna but not radiated as described by antenna's pattern) in the antenna system.

\[ G(\Theta, \Phi) = \frac{\text{power actually radiated by antenna}}{\text{power supplied}} \leq 1 \]

\[ G(\Theta, \Phi) = \zeta \cdot D(\Theta, \Phi) \]
Examples

\[ S_{\text{avg}} = R \frac{A_0 \sin \Theta}{R^2}, \quad S_r = \frac{A_0 \sin \Theta}{R^2}, \quad F(\theta, \phi) = A_0 \sin \Theta \]
\[ F_n(\theta, \phi) = \sin \Theta \]

\[ P_{\text{rad}} = \int S_{\text{avg}} \cdot dS = \int F(\theta, \phi) d\Omega = \int \int A_0 \sin \Theta \sin \Theta d\theta d\phi \]
\[ = 2 \pi A_0 \int_0^{\pi/2} \sin^2 \Theta \sin \Theta d\Theta = 2 \pi A_0 (\frac{\pi}{2}) = A_0 \pi \text{W} \]

\[ D(\theta, \phi) = \frac{F_n(\theta, \phi)}{\int \int F_n(\theta, \phi) d\Omega} \]
\[ = \frac{\sin \Theta}{\sin \theta \sin \phi} = \frac{\sin \Theta}{\frac{1}{2} \sin \phi} = 4 \sin \Theta \]

\[ S_{\text{LP}} = \int F_n(\theta, \phi) d\Omega = \int \int \sin^2 \Theta \sin \Theta d\theta d\phi = 2 \pi (\frac{\pi}{2}) = \pi^2 \text{W} \]

\[ D_0 = D(\theta, \phi)_{\text{max}} = \frac{4}{\pi} = 1.27 \quad \text{dB} \]
\[ D_0 = \frac{4\pi}{S_{\text{LP}}} = \frac{4\pi}{\pi^2} = \frac{4}{\pi} \]

\[ S_{\text{avg}} = R \frac{A_0 \sin^2 \Theta}{R^2}, \quad S_r = \frac{A_0 \sin^2 \Theta}{R^2}, \quad F(\theta, \phi) = A_0 \sin^2 \Theta, \quad F_n(\theta, \phi) = \sin^2 \Theta \]

\[ P_{\text{rad}} = \int F(\theta, \phi) d\Omega = \int \int A_0 \sin^2 \Theta \sin \Theta d\theta d\phi = \int \int A_0 \sin^3 \Theta d\theta d\phi = \frac{8\pi}{3} A_0 \]

\[ S_{\text{LP}} = \int F_n(\theta, \phi) d\Omega = \int \int \sin^3 \Theta d\theta d\phi = 2 \pi (\frac{4}{3}) = \frac{8\pi}{3} \quad \text{W} \]

\[ D(\theta, \phi) = \frac{F_n(\theta, \phi)}{\int \int F_n(\theta, \phi) d\Omega} = \frac{\sin^2 \Theta}{\frac{1}{2} \sin^3 \phi} = \frac{3}{2} \sin^2 \Theta \]

\[ D_0 = \frac{3}{2} = 1.5 = 1.76 \text{ dB} \]

\[ F_n(\theta, \phi) = \sin^2 \Theta \text{ is more directive than } F_n(\theta, \phi) = \sin \Theta! \]

(see Fig 2.8)
Figure 9-11: The pattern solid angle $\Omega_p$ defines an equivalent cone over which all the radiation of the actual antenna is concentrated with equal intensity equal to the maximum of the actual pattern.

Figure 9-12: The solid angle of a unidirectional radiation pattern is approximately equal to the product of the half-power beamwidths in the two principal planes; that is, $\Omega_p \approx \beta_{xz}\beta_{yz}$. 
Figure 2.8 Three-dimensional radiation intensity patterns.