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Leaf Transpiration

$$\lambda E_{leaf} = \lambda g_v (C_{vs} - C_{va}) = \lambda g_v \frac{e_s(T_L) - e_a}{p_a}$$

$$= \lambda g_v \frac{e_s(T_L) - e_s(T_a) + e_s(T_a) - e_a}{p_a}$$

$$= \lambda g_v \frac{e_s(T_L) - e_s(T_a)}{p_a} + \lambda g_v \frac{e_s(T_a) - e_a}{p_a}$$

$$\simeq \lambda g_v s(T_L - T_a) + \lambda g_v \frac{D}{p_a} \quad (3)$$

$D \triangleq e_s(T_a) - e_a$ = vapor pressure deficit

$\Delta = \frac{\partial e_s(T_m)}{\partial T} = \frac{e_s(T_L) - e_s(T_a)}{T_L - T_a}$

$\frac{e_s(T_L) - e_s(T_a)}{p_a} = \frac{\Delta (T_L - T_a)}{p_a}$

$s \triangleq \frac{\Delta}{p_a}$

Recall: $T_L = T_a + \frac{\gamma^*}{s+\gamma^*}\left\{\frac{R_{ni}}{g_{Hr} c_p} - \frac{D}{p_a \gamma^*}\right\}$

and then

$$T_L - T_a = \frac{\gamma^*}{s+\gamma^*}\left\{\frac{R_{ni}}{g_{Hr} c_p} - \frac{D}{p_a \gamma^*}\right\} \quad (4)$$

Combining (3) and (4) and doing some algebra:

$$\lambda E_{leaf} = \frac{s}{s+\gamma^*} R_{ni} + \frac{\gamma^*}{s+\gamma^*} \lambda g_v \frac{D}{p_a} \quad (5)$$

diabatic latent heat loss (water evaporating from stomata due to energy supplied by R_{ni})

adiabatic latent heat loss (water evaporating from stomata because atmosphere is "thirsty" as quantified by D)

where $s = \frac{\Delta}{p_a}$, $R_{ni} = R_{abs} - \epsilon_L \sigma T_a^4$ = isothermal net radiation

$\Delta = \frac{\partial e_s(T_m)}{\partial T}$, $T_L < T_m < T_a$

$\gamma = \frac{c_p}{\lambda}$, $\gamma^* = \gamma \frac{g_{Hr}}{g_v}$

weighting factors

$\frac{s}{s+\gamma^*}$, $\frac{\gamma^*}{s+\gamma^*}$

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What is γ?

$[c_p] = \frac{J}{mol \cdot K}$ $[\lambda] = J/mol$

$\gamma = \frac{c_p}{\lambda}$ is called the thermodynamic psychrometer constant.

$$\gamma = \frac{\text{energy needed to raise temp of mole of air 1 K}}{\text{energy needed to evaporate a mole of water}}$$

$= 6.64 \times 10^{-4}\ K^{-1}$ @ $T = 20°C$

small since much more energy needed to evaporate mole of water — ask

Recall $C_{mol,v}$ = water vapor mole concentration $= \frac{\text{mol vapor}}{\text{mol air}} = \frac{e_a}{p_a}$ — Dalton's law! of partial pressure

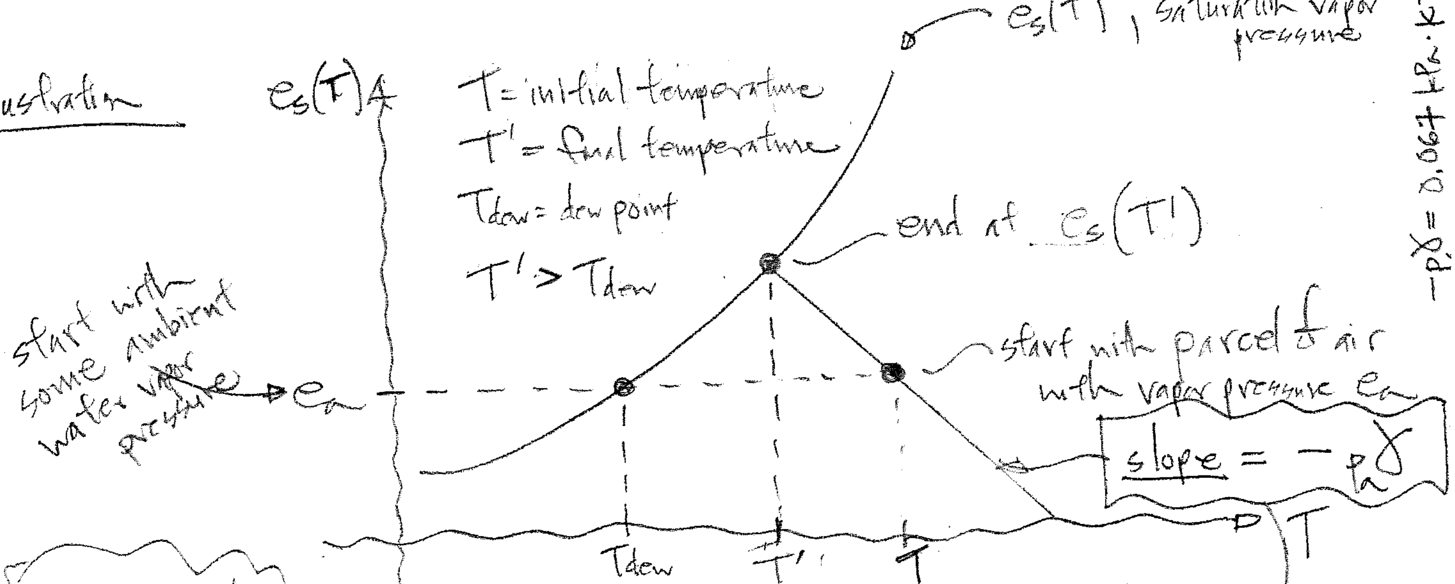
$$p_a\gamma = \frac{e_a}{C_{mol,v}}\gamma = \frac{kPa}{\frac{\cancel{\text{mol vapor}}}{\cancel{\text{mol air}}}} \cdot \frac{\frac{J}{\cancel{\text{mol air}} \cdot K}}{\frac{J}{\cancel{\text{mol vapor}}}},\quad kPa \cdot K^{-1}$$

$$= \frac{J/K}{J/kPa} = \frac{\text{energy needed to raise moist air temp}}{\text{energy needed to evaporate water}}$$

$$-p_a\gamma = \frac{\text{energy taken away (cooling) per K}}{\text{energy needed to evaporate water per kPa}}$$

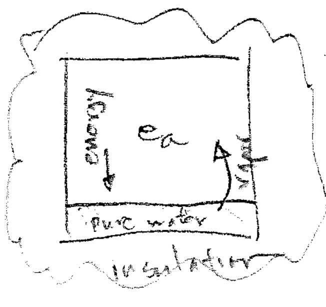
$-p_a\gamma = 0.067\ kPa \cdot K^{-1}$ @ $20°C$

Illustration



water evaporates, moist air cools, continues until saturation reached

rate at which air parcel cools adiabatically!



adiabatic process
(no energy added, no energy taken away)

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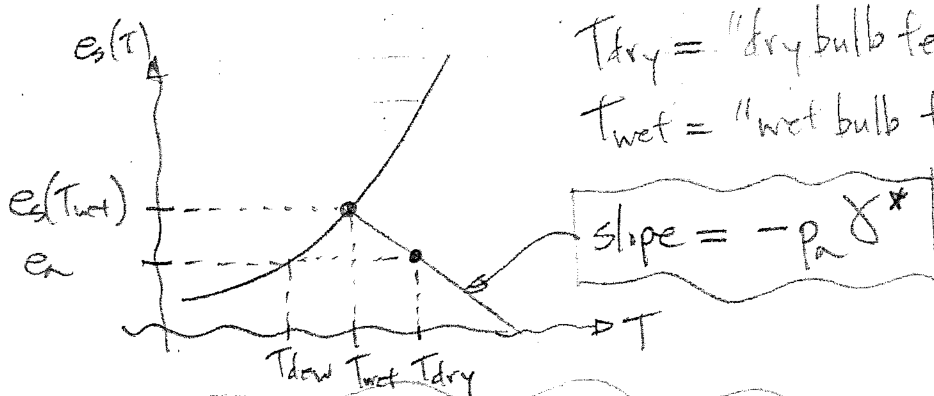
What is γ^*?

$\gamma^* = \gamma \frac{g_{Hr}}{g_v}$ = apparent thermodynamic psychrometer constant

$= \frac{g_{Hr} c_p}{\lambda g_v}$ — for some object w/ (g_{Hr}, g_v)!

$$\gamma^* \frac{\Delta T}{\Delta C_{mol,v}} = \frac{g_{Hr} c_p \Delta T}{\lambda g_v \Delta C_{mol,v}} = \frac{\text{increase in sensible + radiant heat flux}}{\text{increase in latent heat flux}}$$

Illustration



T_{dry} = "dry bulb temperature"

T_{wet} = "wet bulb temperature"

terminology comes from sling psychrometer!

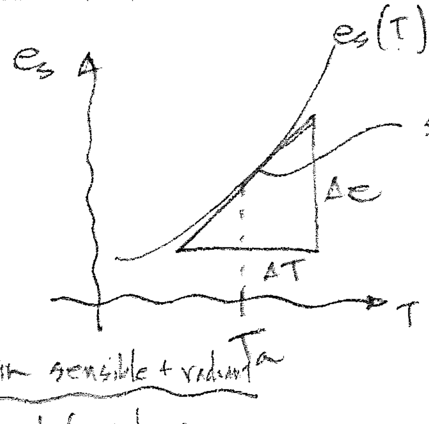
$\boxed{-p_a\gamma^* \text{ is the rate at which an object with a specific } g_{Hr} \text{ and } g_v \text{ would cool adiabatically!}}$

What are the weighting factors $\frac{s}{s+\gamma^*}$, $\frac{\gamma^*}{s+\gamma^*}$?

First note that $\frac{s}{s+\gamma^*} + \frac{\gamma^*}{s+\gamma^*} = \frac{s+\gamma^*}{s+\gamma^*} = 1$

$$\gamma^* \frac{\Delta T}{\Delta C_{mol,v}} = \frac{g_{Hr} c_p \Delta T}{\lambda g_v \Delta C_{mol,v}} = \frac{\text{increase in sensible + radiant heat flux}}{\text{increase in latent heat flux}}$$

$$= \gamma^* \frac{\Delta T}{\frac{\Delta e}{p_a}} = p_a \gamma^* \frac{\Delta T}{\Delta e}$$



slope = $\frac{\partial e_s(T_a)}{\partial T}$ = Δ!

If we are at saturation, $\Delta = \frac{\Delta e}{\Delta T}$ and

$$p_a \gamma^* \frac{\Delta T}{\Delta e} = \frac{p_a \gamma^*}{\Delta} = \frac{\gamma^*}{s} = \frac{\text{allowed increase in sensible + radiant}}{\text{allowed increase in latent heat}}$$

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$$\frac{s}{s+\gamma^*} \cdot \frac{1/s}{1/s} = \frac{1}{1+\frac{\gamma^*}{s}} = \frac{1}{1 + \frac{\text{allowed sens + rad}}{\text{allowed latent}}} \cdot \frac{\text{allowed latent}}{\text{allowed latent}}$$

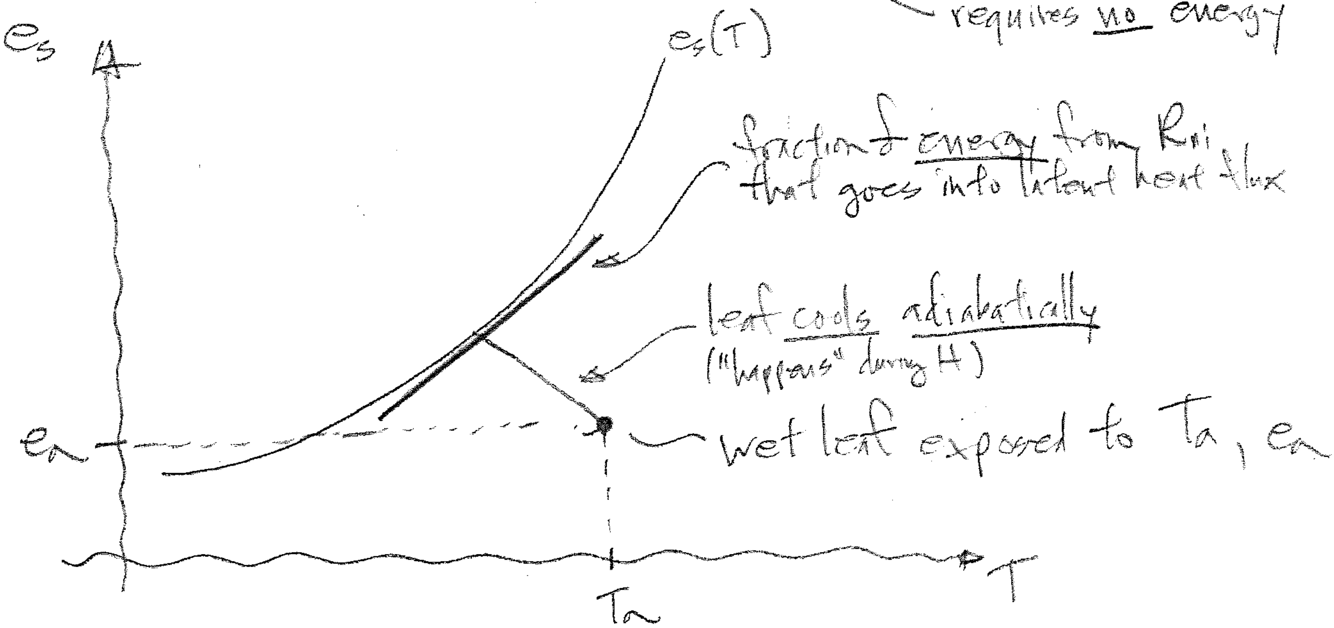
$$\frac{s}{s+\gamma^*} = \frac{\underline{\text{allowable}}\text{ increase in latent heat flux @ saturation}}{\underline{\text{allowable}}\text{ increase in }\underline{\text{total}}\text{ (sens + rad + latent) @ saturation}}$$

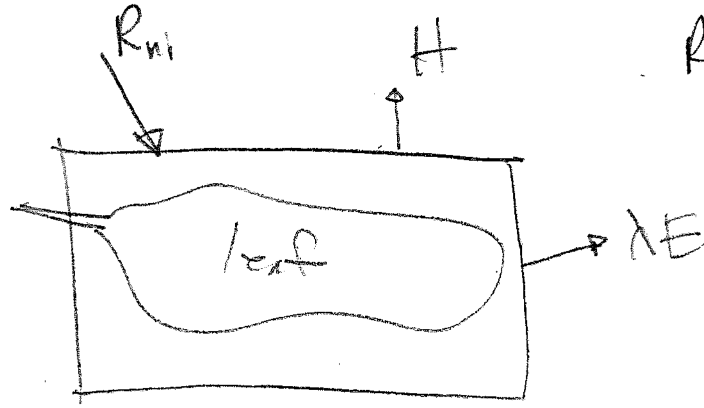
$$\frac{\gamma^*}{s+\gamma^*} = \frac{\text{allowable increase in sensible + rad @ saturation}}{\text{allowable increase in }\underline{\text{total}}\text{ energy @ saturation}}$$

$$\lambda E_{leaf} = \frac{s}{s+\gamma^*} R_{ni} + \frac{\gamma^*}{s+\gamma^*} \lambda g_v \frac{D}{p_a}$$

Penman equation [1948]

requires no energy





$$R_{ni} = H + \lambda E$$

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Canopy Transpiration

Think of an entire plant canopy (community of plants) as one representative "big leaf." Adjust g_v and g_{Hr} to represent the entire canopy, not just one leaf.

$$\lambda E_{canopy} = \frac{s}{s+\gamma^*}\{R_{ni} - G\} + \frac{\gamma^*}{s+\gamma^*}\{\lambda g_v \frac{D}{p_a}\} \quad (6)$$

G = ground heat flux

$\gamma^* = \frac{c_p}{\lambda}\frac{g_{Hr}}{g_v}$, g_{Hr} and g_v are appropriate for a plant canopy

Penman-Monteith equation [1965]

(6) is used to estimate reference evapotranspiration (ET_o).

$$ET_o = \frac{\lambda E_{canopy}}{\lambda} \Big| $$

g_v and g_{Hr} for a 12-cm grass canopy that is "well watered"

$$ET_a = \text{actual } ET = ET_o \times K_c$$

See FAO document on website

where K_c is a crop-specific factor.

"Potential" ET

$$\frac{s}{s+\gamma^*}R_{ni}$$ ← when $D=0$ in (6)

← Priestly-Taylor formula

(I don't completely understand yet)

TABLE 7.2. Integument vapor conductances (from Monteith and Campbell, 1980; Monteith and Unsworth, 1990; and Patten et al., 1988).

Arthropods	**mmol m^{-2} s^{-1}**
Lithobius sp.	32.
Porcellio scaber	13.
Hemilepistus reaumuri	2.8
Glossinia palpalis	1.4
Ornithodorus maubata	0.48
Androctonus australis	0.096
Mammals	
Homo sapiens	5.4
Acomys sp.	2.8
Reptiles	
Caiman sp.	7.5
Terrapene sp.	1.3
Gopherus sp.	0.35

Birds	**mmol m^{-2} s^{-1}**	
Melopsittacus indulatus	4.9	
Excalifactoria chinensis	2.1	
Eggs, several species	0.55	
Vegetables & fruits		
potato tuber	0.77	
apple, Red Delicious	1.2	
apple, Golden Delicious	2.4	
Tomato	5.5	
Orange	5.8	
Radish	275	
Plant leaves	open	closed
Beta vulgaris	260	10
Gossypium hirsutum	375	13
Betulua verrucosa	360	5.9
Pinus monticola	330	17
maize	330	30
soybean	450	40
Quercus robur	41	2.1

also called gvs; "surface conductance" in Chapter 14

or stomatal conductance

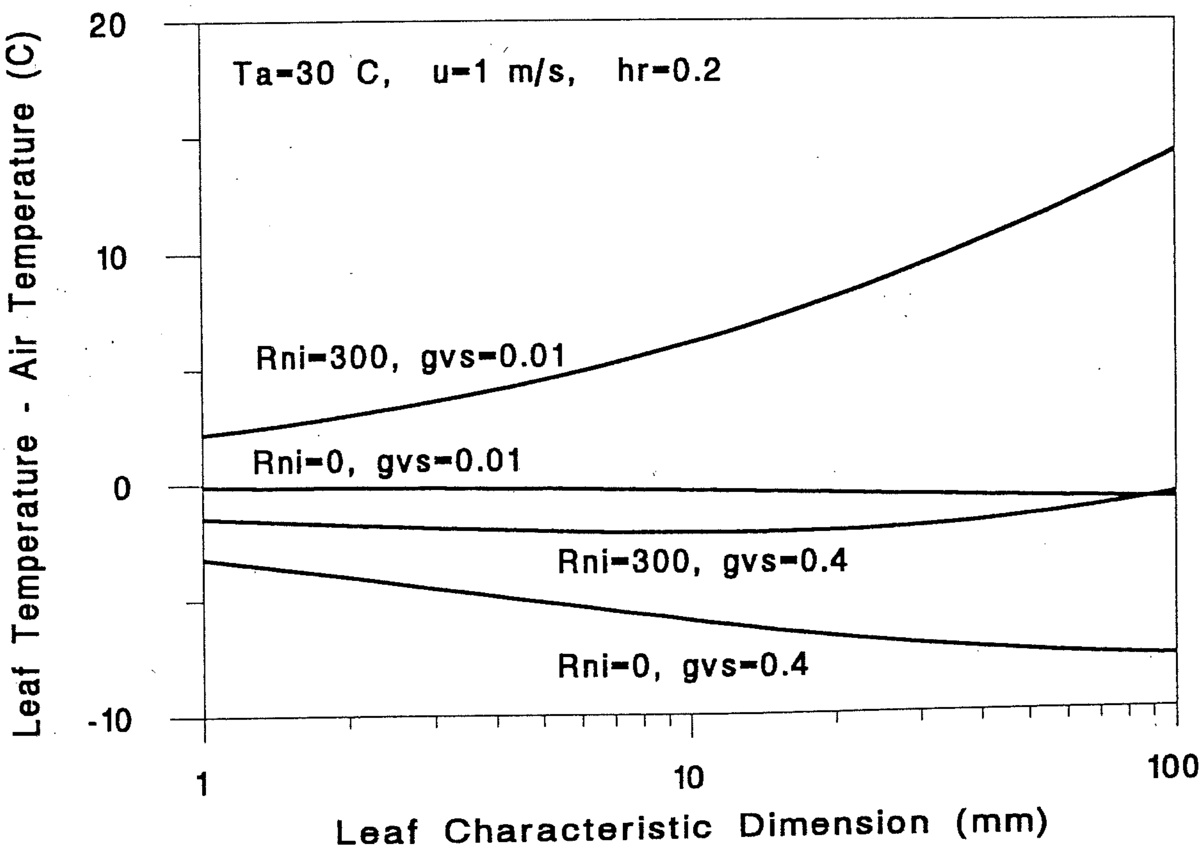


FIGURE 14.1. Difference between leaf and air temperature for various leaf dimensions, stomatal conductances, and radiation loads.

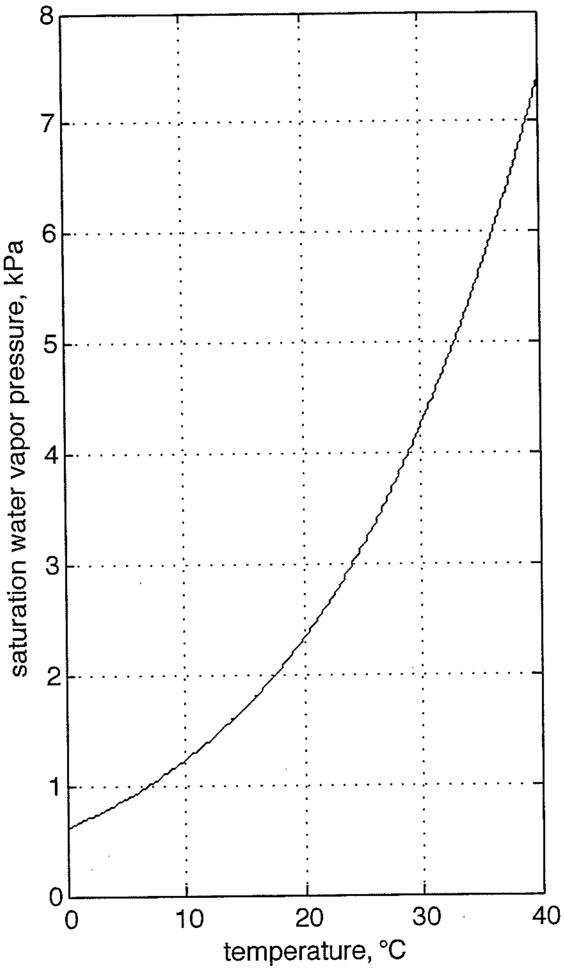
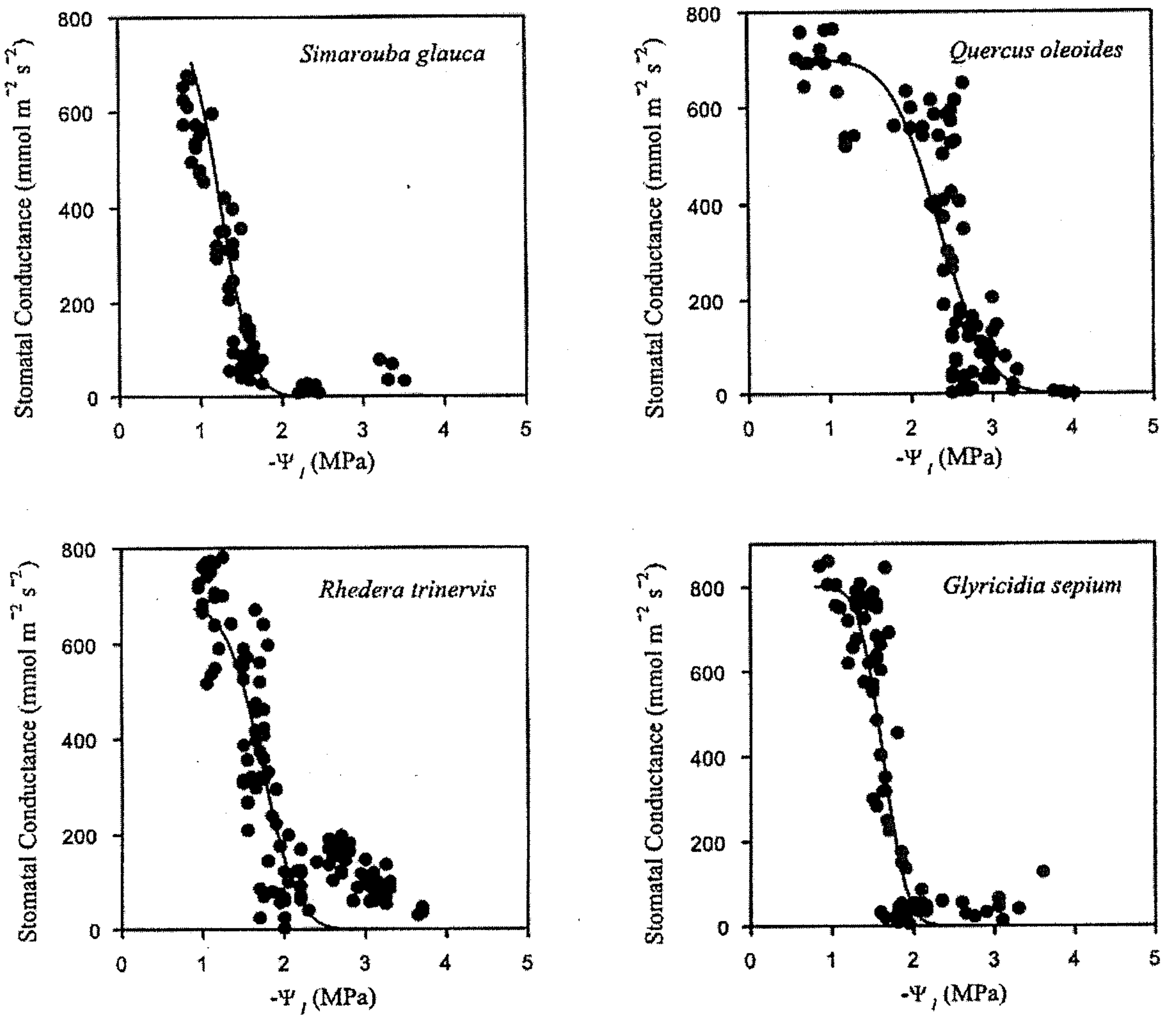


TABLE A3. Temperature dependence of saturation vapor pressure, slope of the vapor pressure function, black body emittance, radiative conductance, and clear sky emissivity.

Temp K	Temp C	$e_s(T)$ kPa	Δ PaC^{-1}	B W m^{-2}	g_r mol m^{-2} s^{-1}	ε_α
268.2	-5	0.422	32	293	0.149	0.66
269.2	-4	0.455	34	298	0.151	0.67
270.2	-3	0.490	36	302	0.153	0.67
271.2	-2	0.528	39	307	0.154	0.68
272.2	-1	0.568	42	311	0.156	0.68
273.2	0	0.611	44	316	0.158	0.69
274.2	1	0.657	47	320	0.160	0.69
275.2	2	0.706	50	325	0.161	0.70
276.2	3	0.758	54	330	0.163	0.70
277.2	4	0.813	57	335	0.165	0.71
278.2	5	0.872	61	339	0.167	0.71
279.2	6	0.935	65	344	0.168	0.72
280.2	7	1.001	69	349	0.170	0.72
281.2	8	1.072	73	354	0.172	0.73
282.2	9	1.147	77	359	0.174	0.73
283.2	10	1.227	82	365	0.176	0.74
284.2	11	1.312	87	370	0.178	0.74
285.2	12	1.402	92	375	0.179	0.75
286.2	13	1.497	98	380	0.181	0.75
287.2	14	1.597	104	386	0.183	0.76
288.2	15	1.704	110	391	0.185	0.76
289.2	16	1.817	116	396	0.187	0.77
290.2	17	1.936	123	402	0.189	0.77
291.2	18	2.062	130	407	0.191	0.78
292.2	19	2.196	137	413	0.193	0.79
293.2	20	2.336	145	419	0.195	0.79
294.2	21	2.485	153	425	0.197	0.80
295.2	22	2.642	161	430	0.199	0.80
296.2	23	2.808	170	436	0.201	0.81
297.2	24	2.982	179	442	0.203	0.81
298.2	25	3.166	189	448	0.205	0.82
299.2	26	3.360	199	454	0.207	0.82
300.2	27	3.564	209	460	0.209	0.83
301.2	28	3.778	220	466	0.211	0.83
302.2	29	4.004	232	473	0.214	0.84
303.2	30	4.242	244	479	0.216	0.85
304.2	31	4.492	256	485	0.218	0.85
305.2	32	4.754	269	492	0.220	0.86
306.2	33	5.030	283	498	0.222	0.86
307.2	34	5.320	297	505	0.224	0.87
308.2	35	5.624	311	511	0.227	0.87
309.2	36	5.943	327	518	0.229	0.88
310.2	37	6.278	343	525	0.231	0.89

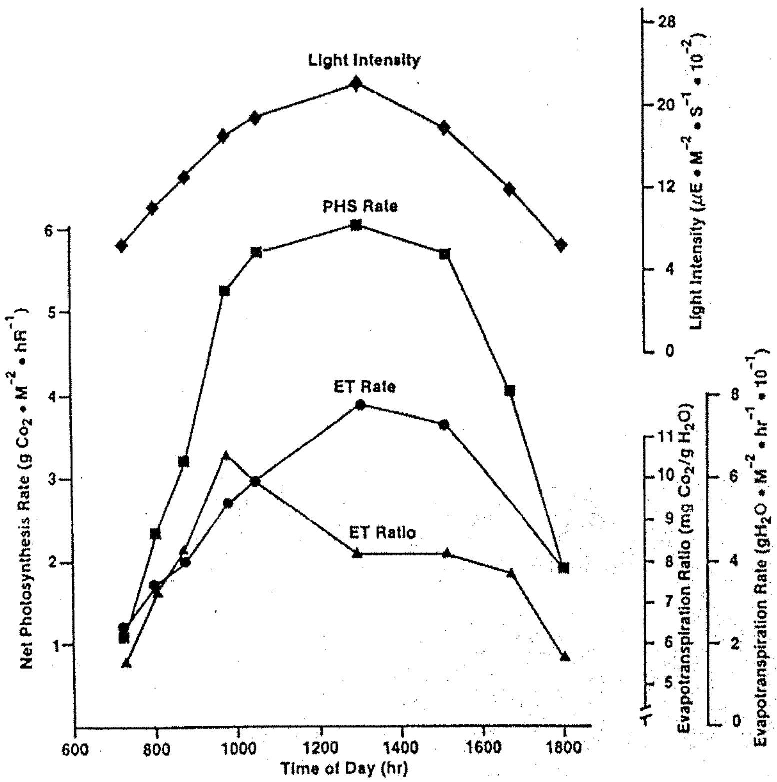
Stomata close when leaf Ψw decreases...



Brodribb and Holbrook, 2003

Diurnal variation of stomatal conductance has direct consequences for leaf and canopy gas exchange

Measure diurnal pattern of transpiration and photosynthesis by a corn canopy



Adopted from Christy, A.L., et al. 1986

Idealized diurnal pattern of stomatal conductance at three levels of water availability