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2001 Mathematical Contest in Modeling (MCM) Summary Sheet

(Please attach a copy of this page to each copy of your solution paper.)

Team Control Number: 260

Problem Chosen: B

Please type a summary of your results on this page. Please remember not to include the name of your school, advisor, or team members on this page.

Summary

We construct an evacuation model, which concludes a series of measures so that the residents on the coast of South Carolina can flee inland to Columbia efficiently.

Firstly, we choose 3 most representative sections of the evacuation route, construct 3 basic models respectively:

1. People evacuating model.
2. Traffic flow model.
3. City congestion model.

Through the three models above we can know that the number of the remaining residents, the traffic flow rate and the maximum capacity of Columbia vary with the passage of time.

Based the three models, we can solve the 6 problems about the evacuation. We focus on the solutions to the two following questions:

1. At a certain time t, how many people are suitable to be allowed to rush onto I-26? That means the evacuation should go on as fast as possible, yet there isn't any traffic congestion on I-26.
2. What is the proper arrangement to avoid Columbia's congestion.

Secondly, we use computer programs to simulate the traffic condition of South Carolina and get the accurate results.

Lastly, we analyze the strength and weakness of our models and the improvement of them.

Restatement of the Problem

In South Carolina, when hurricane alarm is announced the residents of the coastal region evacuate the coastal region. Interstate highway I-26 is the principal road going to the safe inland. But under the circumstances of emergent evacuation, so many vehicles might crowd on the highways almost at the same time. Without proper plan the traffic nightmare occurred. The focus of the problem is to control the highway's traffic flow in evacuation appropriately. There are some strategies which can be adopted. The principal proposal is the reversal of traffic on I-26. We are asked to estimate measures under different conditions, and find solutions to problems.

Assumptions

1. The forecast of hurricane is accurate. There is enough time to make necessary preparations. For instance, people can get the 48-hours forecast of the hurricane.
2. Only through the cities near where they live can the residents drive on the highways. The time they spend driving from their homes to cities can be neglected.
3. Every part of the highway has the same capacity. It means traffic jam only occurs in the suburban region, and could not occur on the midway of the highway.
4. Government decisions are binding to people.
5. There are few vehicles heading for coastal regions before the landfall of the hurricane.

Problem Analysis and Model Structure

We can construct the traffic flow model, the residents' evacuation model and the city congestion model. Furthermore, through traffic flow model, residents' evacuation model and the city congestion model are connected. In that case, we can analyze the 6 questions and draw conclusion.

Model Design

Residents evacuation model

We can construct the model in two different aspects. One is that the residents can evacuate freely. Government policy cannot affect their behaviors. The other is that the evacuation of residents is entirely under the government's control.

Model 1:

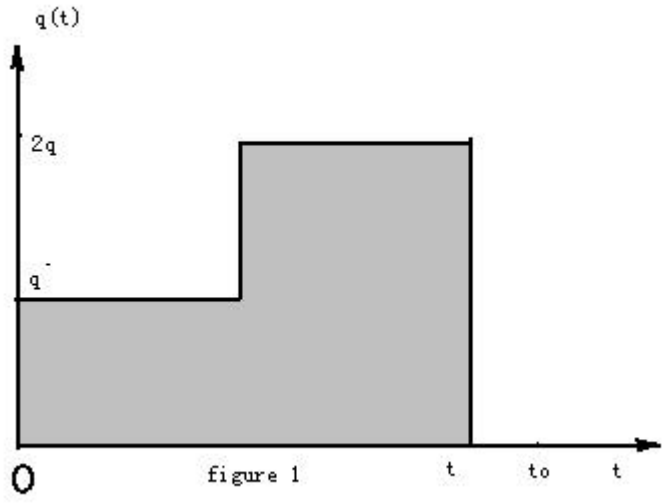
Assumption1, the forecast can give us enough time to escape. So we assume there are few residents evacuating long before or just before the landfall of the hurricane. Most of the residents choose the moderate time to evacuate. According to the law of the phone calling, the number of

evacuees at a unit time conforms to the Possion Distribution with the parameter λ. Considering the impact of the forecast time on residents' psychology, we find the parameter λ is the function of the forecast time. s is the parameter of the forecasting, which is actually forecast time. λ_0 is the parameter which means the forecast can give us enough time to evacuate. We assume $\lambda = \lambda_0 e^{-\frac{1}{s}}$. Now we found it is helpful to avoid the traffic jam, if we can forecast the hurricane as early as possible. But the capability of forecasting the hurricane is restricted by the scientific methods and other conditions. So we put forward and develop model 2.

Model 2:

It is well known that the most effective way of escaping from disasters is that evacuation should be planed well. We assume government is capable of controlling the traffic flow rate q on highways. The key point is to make decision of turning the entire high way into one-way traffic. Assuming $\mathbf{q_{max}}$ is the maximum rate of the traffic flow of the one side of the high way. We get

$q(t)=\begin{cases} q_{\max} & two \quad direction \quad road \\ 2q_{\max} & one-way \quad road \end{cases}$. The number of the vehicles since the time of the announcement of the forecast $N(t)=\int_0^t q(t)dt$ is the area of the shadow part of the figure 1.



Point O is announcement of the forecast point. $\mathbf{t_0}$ is the predicted landfall point of the hurricane. If evacuating successfully there must be $\mathbf{N(t_0)>A}$, A is the number of the evacuating vehicles. In that case, people can avoid traffic jam when evacuate.

Traffic flow model:

In order to make a reasonable plan for the evacuation of the residents, we describe and

analyze the roads' conditions in the coastal region of South Carolina. The following variables are used to describe the conditions of the roads.

1. **k** is the density of the vehicle , it means the number of vehicles in the unit length.**(k=N/l)**. **N** is the number of vehicles on road. l is the length of the road.
2. **u** is the average velocity of vehicles on a certain part of the road.
3. **q** is the rate of the traffic flow. It means the number of vehicles running through a given point of the road in a unit time.

Based on the well-known Greenshields' rate of traffic flow-density of the vehicle correlation :

$$\mathbf{q = ku} \tag{1}$$

and the conclusion of Book 1, we get the equation

$$q=ak\ln\frac{kj}{k} \tag{2}$$

which reveals the relation between the rate of traffic flow and the density of traffic flow.

Based on the equations (1) and (2), We get the equation which reveals the relationship between density of vehicles and the average velocity of the traffic flow:

$$k = k_j e^{-\frac{t}{a}} \qquad (3).$$

Finally, through equations (1) and (3),we get the equation which reveals the relationship between the rate of traffic flow and average velocity, $q = uk_j e^{-\frac{u}{a}}$.

Analyzing the derivative of equation (4), we know when **u=**$\sqrt{a}$, the maximum rate of traffic flow is $q_{max} = \sqrt{a}\ k_j e^{-\frac{u}{\sqrt{a}}}$. That is the key value in our model.

Where:

$\mathbf{k_j}$ is the maximum density of the vehicle, it means there is no clearance between the vehicles on road.

m stands for the average length of vehicles. We get the equation $\mathbf{k_j = 1/m}$.

a is the carriage capacity of the road: it is a constant which reveals quality of the road itself. We can get this value by the observation on the spot or by using equation (2).

City Congestion Model:

Each city has capacity for its own vehicles to drive in and drive out. But more important is that each city has capacity for vehicles from other regions.

$\mathbf{q_0}$ is the traffic flow rate of riding out of the city.

$\mathbf{q_i}$ is the traffic flow rate of riding in the city.

Q is the capacity for vehicles coming from other regions.

$$Q \geq \int_0^t (q_i - q_0)dt \quad .$$

If the correlation equation is established, city congestion will not occur. If not, city congestion will occur.

Solutions to the Problems

Assuming that there are no vehicles riding in from other states.

First, we should estimate the numbers of vehicles driving through Charleston and Myrtle Beach respectively: the number of evacuees escaped from Charleston is about 680,000, that from Myrtle Beach is about 270,000. Assuming that each vehicle holds 4 persons, then there will be about 70,000 vehicles driving through Charleston and 67,500 vehicles driving through Myrtle Beach. If the number of vehicles on I-26 increases, it will have impact on the Columbia's traffic. Further more, it will have impact on the rate of traffic flow on I-26. We should consider the fact: if government turn the coastal-bound lanes of the I-26 into Columbia-bound lanes in advance, the traffic flow on the north part of I-26(from Columbia to northwest) will be affected, so that the northern part of I-26 has to be turned into one-direction road in advance as well. It is no good for Columbia's traffic. It is vital to choose the right time for turning the coastal-bound lanes of I-26 into Columbia-bound lanes.

Assuming that:

1. The cities located to the northwest of the Columbia have enough capacity for the ride-in vehicles.
2. The capacity for vehicles of Columbia is **Q**.
3. The rate of traffic flow on the two-lane road is $\mathbf{q_{max}}$, both in Charleston and Columbia.
4. The forecast of landfall of hurricane gives us **T** hours to evacuate.
5. Analyzing the traffic flow model, residents evacuation model, and city congestion model, we find if residents evacuate timely and avoid the traffic jam at the same time, the following formulas must be established.

$$\begin{cases} \int_0^T q_1(t)dt \geq A & Charleston \\ \int_0^T q_2(t)dt \geq B & MyrtleBeach \\ \int_0^T q_3(t)dt \leq Q & Columbia \end{cases}$$

We get the results of the above formulas by approximately estimating. Under the normal circumstances, there is always 48 hours forecast, the maximum rate of traffic flow $\mathbf{q_{max}}$ is 3,000 vehicles per hour. So the residents around Myrtle Beach can evacuate completely in 22.5 hours. The highway from Myrtle Beach to Columbia need not be turned into one-direction road. Since Columbia has three highways for leaving the city, there are almost no vehicles at a standstill in Columbia, when I-26 is two-direction road.

In order to reduce the impact on Columbia's traffic, we suggest that at the beginning, the residents of Charleston and Myrtle Beach both evacuate at the maximum traffic flow rate. When Myrtle Beach finishes its evacuation, residents of Charleston should double the traffic flow rate. The time t Charleston needs to evacuate all the residents can be got as follows:

$$t = \frac{A - B/q_{\max} \cdot q_{\max}}{2q_{\max}} + B/q_{\max} = \frac{A+B}{2q_{\max}} = 40(hours) \le 48(hours)$$

If this plan is adopted, the residents of coastal region can evacuate in time, and the congestion of Columbia will not occur.

Computer Simulation

Overview of the Program

This program is based on the following principles:

1. The rate of traffic flow of the roads with same lanes is fixed.
2. When the number of remaining residents in one coastal city drops to zero, the other city will double the flow rate.
3. The total population is fixed, so the reduction of the population of one city equals to the increase of the other city.
4. The inland city Columbia is able to release the stress of the evacuation flow timely.

The data input:

1. the rate of traffic flow released from each city.
2. the period of time from the forecast to the arrival of the hurricane .

The simulative functions that have been implemented:

It outputs the chart indicating the relations between the population of each city and the rate of traffic flow on the certain road, from which we can observe when the total population is able to evacuate.

Using of the program

We use a computer program to testify the model and the results. Here is our thought. At first, we built a topological structure of the traffic network, initiate the maximum rate of traffic flow on each road, the population of each city and the capability for accommodation. At second, we transferred residents from the coastal region to Columbia at the maximum rate of traffic flow. Here are three judgments:

1. Whether the Columbia has already made full use of its capability to accommodate evacuees and the population is beyond its capability.
2. Whether residents can evacuate timely at the given rate of traffic flow.
3. Whether to turn the highway into one-direction road.

The program simulates the traffic condition dynamically and shows the variable conditions at different moments. We have enclosed the Pascal program and the outcome of the program. The conclusion is that we should turn I-26 into one-direction road 23 hours after the beginning of the evacuation.

Concerning the question 1, the outcome of the computer simulation verified our theoretical analysis. Now, we use theoretical model to make qualitative analysis, and use simulation model to make quantitative analysis for the following questions.

Concerning the question 2, according to the meteorological files, hurricanes land on the southwestern coastal region. If we adopt the strategy of staggering evacuation, the residents living in the southwestern region should evacuate firstly. At that time, since there are no evacuees on the road of Myrtle Beach, I-26 can be turned into one-direction road for evacuation. After the residents in southwestern finish their evacuation, Myrtle Beach can reverse traffic as well. This plan makes full uses of time and the carriage capacity of the roads.

Outcome of the simulation:

The time needed to complete the evacuation T=39.5 hours.

Concerning the question 3, if we make full use of the smaller highways, the maximum rate of traffic flow on every road will rise. It will speed up evacuation.

Assume traffic flow rate on all small highways is 3,000 vehicles per hour.

Outcome of the simulation:

The time at which period the road is turned into one-direction T_0=17 hours.

The time needed to complete the evacuation T=30 hours.

Concerning question 4, if we establish temporary shelters in Columbia, the capacity of accommodation will increase. It will put off and reduce the time of reversal of the traffic on I-26.

Assume Columbia city can hold at most 600,000 people.

Outcome of the simulation:

The time needed to complete the evacuation T=35.5 hours.

Concerning question 5, if there are no restrictions on the length of vehicle, the maximum density of vehicle flow k_j will be reduced by those too long vehicles. Since $q_{max} \propto k_j$, q_{max} will be reduced as well. It will the slow down the evacuation. If there is no restrictions on the number of vehicles brought along. The numbers of evacuating vehicles will increase. It will also slow down the evacuation.

Assume the average length of vehicles decrease by 10% and each vehicle holds one more person.

Outcome of the simulation:

The time needed to complete the evacuation T= 28.5 hours.

Concerning the question 6, considering the residents of Georgia and Florida pouring into the south region. The number of evacuating residents will increase. It takes more time for evacuation.

Assume the number of evacuees coming from Georgia and Florida is 100,000.

Outcome of the simulation:

The time needed to complete the evacuation T=52.5 hours.

Model Improving

We can get more precise results if we improve the following points:

1. We can consider the situation that the transportation ability of the highway is different from one another and the different sections of the road have various traffic flow qualities.

2. Also if the velocity and the route of the hurricane is learned, we can arrange the evacuation in more efficient order.
3. We can draw a figure that reflects the density of population distributing.

References

[1] W.D.Aston, The Theory of Road Traffic Flow, Methuen, London, 1966.
[2] John L. Casti, 1996, Would-be worlds: How Simulation is Changing the Frontiers of Science. Shanghai Scientific And Technological Education Publishing House.
[3] Zhejiang University,Probability Theory, Higher Education Press,China.
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News Report

Fleeing quickly: an evacuation plan for South Carolina

The nightmare of the hurricane attacking South Carolina might not have happened, if we had made a good plan for that when evacuating.

Some students did some study on that disaster and had computer programs simulate it. They concluded that if the hurricane comes again in the future, people can efficiently evacuate from the hurricane if adopt their strategy. Their study indicates that, if people follow the command of the government entirely and make full use of the highway, the evacuation will be completed in 40 hours. So evacuees are required to be in order so as to evacuate promptly from the coastal area. Moreover, more shelters built around the city of Columbia will contribute to the release of the traffic flow so that the vehicles could run to the safe area to the northwest to Columbia freely. They also suggested people driving vehicles that are short but capable of holding many people. For example, average speaking, every 4 people taking a 4-meter- long car would complete the evacuation in 40 hours while every 5 people taking a 4-meter-long car will only take 29 hours. And also, less packages should be taken. In 1999, some of the coastal residents of Georgia and Florida, who were fleeing the earlier predicted landfalls of Hurricane Floyd to the south, came up I-95 and compounded the traffic problems of our state. This added about 12 hours to the time needed in our evacuation. So we are suggested to have the same timely prediction as the two states in order to start our evacuation as early as possible.

Remember, do not panic and follow the command of the government before hurricanes. Thus the disaster in 1999 will never recur.