1999 年 MCM 题 A

合肥工业大学 杨友庆、谢建生、程雪林

1999 年 2 月 1 日-8 日

Restatement of Problem

For some time, the NASA has been considering the consequences of a large aster- oid impact on the earth.We are to assume that an asteroid is on the order of 1,000 m in diameter, and that it strikes the Antarctic continent directly at the south Pole.

Now we are asked to consider the effects of this impact,and concerns that an impact there could have considerably different consequences than one striking elsewhere on the planet.Then we estimate the following questions:

- the amount and location of likely human casualties from this impact.
- the damage done to the food production regions in the oceans of the southen hemisphere
- possible coastal flooding caused by large-scale melting of the Antarctic polar ice sheet

Assumptions

- There are 2,700 meters in average thickness of ice around the South Pole.
- The large asteroid is a sphere with even density.
- The diameter of asteroid that doesn't go into atmosphere is 1,000 meters.Because there is seldom air on 70 kilometers high,we can assume that the initial height is there. Moreover it points directly at the South Pole in the beginning.
- It only is attracted by the pull of the earth, and any other sphere has no effect on it.
- It strikes into the ice, and the energy explosion is divided into two parts:one is used to melt the ice,the other to produce earthquake wave which can cause tsunami.
- Provided the average temperature of ice is 30 below zore.and heat energy only melt them to zero.
- We don't consider the moving iceberg caused by the earthquake.
- The food production regions concern only the fishing ground at the southern hemisphere and the regions around Antarctic continent.
- The initial velocity is from 14 to 18 kilometers per second. It can come from the solar system and attracted by the earth.
- We haven't consider the rotation of the earth because the asteroid impacts on the South Pole directly

Problem Analysis

In analyzing the procedure of the asteroid striking the South Pole,we can divided it into three stages:

The first stage:It turns into fiery sphere,falling from the height of 70 km at the speed of 15 km per second.There is about 3,200 degrees centigrade at the head of it. The relationship of characteristic length and characteracteristic time is $q \propto \frac{r c_v l}{k}$. According to what we know,the separation of asteroid needs about ten seconds,but the asteroid given flies for less than five seconds,so it can't explode and will strike the ground directly. Moreover the mass of it will decrease about 20% because of fire.

The second stage: It dashes into the ground and explodes below. Its kinetic energy changes into thermal energy partly to melt the ice,and the remains can cause earthquake.

The third stage:Mixed wave of sexism at the speed of from 3 to 4 kilometers per second transmits all round at the earth's surface by A=A(x,t)expi(kx-wt).According to experiential formula,we can determine the grade of the earthquake at the South Pole and the scope of destruction.

When the earthquake wave transmits along sea floor,it can produce tsunami.Its intensity is related to the extent of movement caused by earthquake wave.

The thermal explosion energy melts the ice sheet into water that pours into oceans near the Antarctic. It will make the sea level rise so that many countries of the southern hemisphere are destroyed by coastal flooding. We estimate the destroyed extent of agriculture and destroyed city along the sea is related to the rising height of sea level.

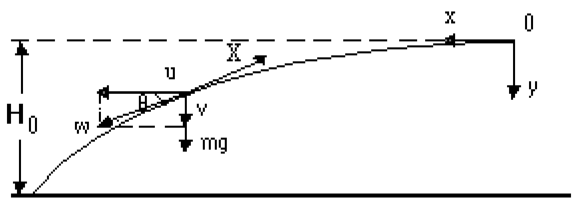
Shock wave and the rise of temperature begin from the South Pole,effecting the global climate gradually.The change of climate leads to the change of environment where living beings exit in the oceans,so the fishery will be effected.

Model Design

From the above ,we build different mathematical models in the three different procedures.

1. Model of Falling in the Atmosphere

In the plane of the earth's axis and the asteroid,we set X-Y coordinate .Coordinate pot is at the 70 km height.It is effected by gravity,wind-force and air restistance.For its mass is very huge,the effect of wind-force can be neglected.(about the analysis of field of force when flying see figure 1).Its moving satisfies the following basic equation:



$$m(t)\frac{du}{dt} = -X\cos q$$

$$m(t)\frac{du}{dt} = m(t)g - X\sin J$$

$$\frac{dx}{dt} = u \qquad \frac{dy}{dt} = v$$

$$w = \sqrt{u^2 + v^2} \qquad \tan(q) = \frac{v}{u}$$

u is the vertical component of velocity of the asteroid.(m/s)

v is the horizontal of velocity.(m/s)

m is quantity of asteroid,it changes by m(t)=m_0-kt in which m_0 is the initial quantity.

x is the wind-force(N).its expression is $x = \frac{1}{2} r(u^2 + v^2) C_d dS$ in which c_d is the resistive cofficient,and its expression is

$$\begin{cases} 0.95 & m > 1.3 \\ 0.75m - 0.025 & 0.7 < m < 1.3 \\ 0.5 & m < 0.7 \end{cases}$$

a is the velocity of sound,equal of 340m/s.

m is equal to m=$\frac{|w|}{a}$.

s is the area against the wind(m^2) equal to pR^2 (R is the radius of asteroid).

r is the air density at the height (m^2) of H(H is equal to H=H_0 - y in which H_0 is 70 km, and y is the vertical coordinate)(kg/m^3). Its similar expression is $r = r_0 e^{-\frac{H}{7000}}$,and in it r_0 is the standard density of air equal to .25 kg/m^3

w is the asteroid's fly speed (m/s).

q is the angle between x and the ground.

From it we can get the velocity,quantity and the kinetic energy when it flies to the ground.

Here we don't consider the velocity of the wind .In fact it is related to the height, and its expression is

$$v = \begin{cases} 3 + 0.005H & H < 10^4 m \\ 80 - 0.0027H & 10^4 < H < 3\times10^4 m \\ 0 & H > 10^4 m \end{cases}$$

But because the quantity of the asteroid given is very huge, the effect of wind can be neglected.

Model of Strike

It strikes the ice sheet with high speed. It will go forward in the ice and explode at the depth of h_0 .We can determine it is decided by kinetic energy of asteroid at this time and physical quality of the ice sheet. We find:

$$h_0 = \left(\frac{2m_0}{C_x sr} \ln \frac{u_0}{u_k} \right) \cos q$$

m_0 is the mass of the asteroid

u_0 is its velocity.

C_x is the resistive velocity.

r is the density of the asteroid.

q is the angle of direction of its velocity and the face of shock wave.

The depth caused by the shock wave is expressed as below, Furthermore explosion deepens the crater and makes it a taped crater. it is decided by the shock energy.

$$h^* = \sqrt[3]{\frac{3M_0 u_0^2 h}{8Qpr}}$$

h is the utilization coefficient of energy.

Q is the energy density which can destroy the ice sheet, and its expression is $Q = \frac{u_k^{*2}}{2}$

(u_k^{*2} decided by the quality of ice sheet is equal to 2 km/s).

$\bar{h}$ is equal to 3.05 here.

Now the total depth of crater can be got $h = h_0 + h^*$

At last we can calculate the assign of the energy because of explosion.The energy used to cause earthquake is $E_h = m_0 \left(\frac{p_h (v_0 - v_h)}{2} - e_k^* \right)$

$$m_h = p_h (v_0 - v_h) = \left(\frac{u_0}{2} \right)^2 \qquad e_k^* = \frac{u_k^{*2}}{2}$$

So we can get $E_h = \frac{1}{2} m_0 \left[\left(\frac{u_0}{2} \right)^2 - u_k^{*2} \right]$ and the energy used to melt the ice sheet is $Q = hE_0 - E_h = \frac{1}{2} m_0 u_0^2 h - E_h$

Model of Effect

A. Model of Destruction of Earthquake Wave

The center of the explosion can be looked as the source of earthquade.Its wave attenuates all around by A=A(x,t)expi(kx-wt).Provided that attenuation is same at any direction,concentric spherical surface will be formed

Some of E_h can be transmitted into atmosphere. h' is the utilization coefficient and E_s is the energy transmitted into the earth's crust equal to $E_s = h' E_h$,so we can get the energy level equal to $K = \log E_s = a + bM_s$ [2]

Referring to the Bath's data we can get the equation

$$\log E_S = 5.29 + 1.43 M_S$$

According to experiential formula

$$M_B = 2.5 + 0.63 M_S \quad [3]$$

$$M_B = 1.7 + 0.8 M_L - 0.01 M_L^2 \quad [3]$$

thus we get the M_L .In them

M_S is the earthquake level of surface wave.

M_B is the earthquake level of body wave.

M_L is the Richter's earthquake level.

Then from the Toppozada's equation $M_L = 0.86 + 1.09 \log S_{5m}$. in which S_{5m} is the area where earthquake level is beyond five.So we can get the numerical value of S_{5m}. At these places, people can feel earthquake obviously and the buildings can be destroyed seriously.By it we can get the scope of destruction.

B. Rise of Sea Level

For the thermal conductivity of ice is 2.5,it can be looked as thermal insulation material.Moreover the snow's absorptivity is 0.985 , so we can assume that ice absorbs total quantity of heat ,making $-30^o C$ ice into $0^o C$ water.Through the crack produced by earthquake water pours into the ocean.Specific heat $\bar{c}$ is the average value from $-30^o C$ ice into $0^o C$ water.

$$Q = \bar{c} \Delta T m + q m \qquad m = \frac{Q}{\bar{c} \Delta T + e} = rV$$

ΔT is changes of temperature.

q is the heat of fusion.

m is the quantity of water.

r is the density of water.

v is the volume of water.

According to v we can calculate the height $h = \frac{v}{s}$ in which s is the area of total ocean.

C. About Tsunami

As the most area of the South hemisphere is ocean,the energy of the earthquake causes huge tsunami. The greatest height of the tsunami is attenuate with the increase of spreading distance.Now we have the formula of the height as follows:

$$y_s = \frac{hR}{rk\cos(2pd/L)}\sqrt{\frac{kC_G}{-kC_G^{'}}}\cos 2p\left(\frac{t}{T}-\frac{r}{R}\right)J_1\left(\frac{2pR}{L}\right)$$

$$C_G = \sqrt{gd}\left(\frac{1}{2}\sqrt{\frac{\tanh kd}{kd}}+\frac{1}{2\cosh^{\frac{3}{2}}kd}\sqrt{\frac{kd}{\sinh kd}}\right)$$

$$C \approx \sqrt{gd}$$

k is the number of waves.

h is the amplitude of some part of sea-bottom moving vertical suddenly.

R is the radius of sea-bottom moving vertical suddenly.

r is the distance from the source.

t is a single wave's period.

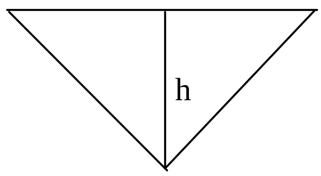
l is a single wave's wavelength.

c is the phase velocity of the wave.

$C_G^{'}$ is the derivation of k.

$J_1(x)$ is the first basal function

in it the paramater.h equal to A can be got from the following formula:



$$M = \log(A/T) + 1.66\log\Delta + 3.30$$

A is the maximum amplitude of face wave.

Δ is among $25^0 C$ and $140^0 C$

Solving the Model

The first stage:asteroid's moving in the atmosphere.

We assume that its quantity would decrease by linear equation.

$$q = 90^o c \quad v_o = 15km/s \quad m_o = \frac{1}{6}pd^3 = \frac{p}{2}\times 10^{12} \text{ kg}$$

Inputting the above constants into our model game is quite accurate results

$$m_1 = \frac{4}{5}m_0 = \frac{2p}{5}\times 10^{12} \text{ kg} \quad v_1 = 14.8km/s$$

At the procedure frictional does negative work which changed into thermal energy.Futhermore quantity was decreased because of fire.Concerning them, the temperature of atmosphere would rise a little.Provided that it released

From $\Delta E_k = mgH - w_f$ we can get $Q = w_f = mgH - \Delta E_k = 4.0\times 10^{19}$ J

The second stage:asteroid's striking.

Given the following

$$m_0 = \frac{2}{5}\pi \times 10^{12} kg \quad C_x = 3.8$$

$$\rho = 3 \times 10^3 kg / m^3$$

$$S_1 = \frac{4}{5}S = \frac{1}{5}\pi d^2 = \frac{\pi}{5} \times 10^6 m^2 \qquad u_0 = 14.8 km/s \quad u_k^* = 2 km/s$$

we can get $h_0 = 653m$ $h^* = 1537m$ $h = h_0 + h^* = 2290m$

the results of the energy turned into kinetic and the energy turned into thermal energy are as follows

$$E_h = \frac{1}{2}m\left[\left(\frac{u_0}{2}\right)^2 - u_k^{*2}\right] = 3.19 \times 10^{19} J$$

$$Q = \eta E_0 - E_H = 7.81 \times 10^{19} J$$

The third stage: the explosion effected the global.

A) Scope of destruction caused by earthquake.

$$E_S = \eta' E_H$$

from it when u_0 =14.8 km/s . we can get the results as follows when η^i was different.

item	value					
η'	1	0.9	0.8	0.7	0.6	0.5
S_{5m} $(\times 10^8 km^2)$	3.022	2.814	2.599	2.376	2.142	1.695
percentage of global (%)	59.30	55.17	50.97	46.58	42.00	37.15

Table1

Make the further step,we can see the relation between S_{5m} and η' (see the table1) . The figure about the scope of damage is added to appendix

When $u_0 = 17.8 km/s$ the results outlined in the table 2 was

item	value					
η'	1	0.9	0.8	0.7	0.6	0.5
S_{5m} $(\times 10^8 km^2)$	3.941	3.661	3.387	3.094	2.789	2.466
percentage of global (%)	77.27	71.94	66.42	60.67	54.68	48.35

Table2

B) Rise of sea level

According to

$$Q = 7.81 \times 10^{19} J \quad \Delta T = 30k \quad q = 333.55 \times 10^3 J / kg$$

using the formula given front we can get the result $h = 2.3\times10^{-4}m \approx 0.23mm$ from it we can see the height increased wasn't obvious,so we could judge that there was little effect on the ocean.

C) The effect of tsunami

The other parameters can be determined by the energy of the earthquake.According to the program ,we find that:

When r is 4,500 km, namely South latitude 49.5^0C , $y_s = 9.2m$.

When r is 6.500 km ,namely South latitude 36^0C , $y_s = 4.7m$.

Result Analysis

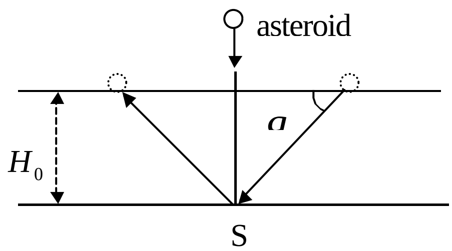
1 We consider there is an angle $q(\geq 50^0)$When the asteroid enters the atmosphere the picture shows: if q is small, the road of the asteroid moving in the atmosphere is long .But the change of the asteroid's speed also is little.So we think that the kinetic energy of the asteroid near the south has no relation with q .But q will influence the depth of the hole.

2.The asteroid moves 653 meters in the ice sheet before its explosion,so we think sixty percent of the its energy enters the ice sheet .We think it reasonable for the earthquake influencing the human that h^i is 0.6.At this time,the damaged area the intensity above the fifth grade will be 2.142 X $10^{14}m^2$,it is at the south of 14.4 degrees south latitude.The countries near the South Pole for example Argentina,Chile,New Zealand,Australia and South Africa are damaged seriously.We estimate that the number of the dead people will be amount to 10 million ,and the injured people will be 100 million. .

3.When the asteroid passes through the atmosphere,the heat that it produces will be 4.0 X10^{19} J

The heat will form a large warm air mass.The warm air mass will move to the equator pulled by the cold air climate.It will influence the existence of the whales and shrimps near Peru Fishery.So we estimate the proportion of the damaged fishery will be $1/3$.

4.By the data we calculated, it shows that the rising of the sea level is very minor.So,we think that the rising caused by melting the ice will hardly effect the human-beings.



5.The tsunami will cause a tremendous damage to the coastal countries.At the Chile and southern Argentina the tsunami will have 9.2 moaned at Australia and South Africa the tsunami will have 4.7m. It will destroy coastal cities of the countries seriously.

Model Improving

We can get the more precise result if we improve the following points:

1.Distribute the impacting energy more reasonably and consider the mutual influence between the heat absorbed by the atmosphere and the heat emitted by explosion. .

2. According to the state of the human distribution in the affected countries and the previous data about their damage,we can estimate the number of the dead and injured people and the affected area more precisely.

3.We should consider the influence of the other food production in the oceans of the southern hemisphere and give the more reasonable estimate.

Strengths andWeakness of the Model

Strengths

1. Our model is divided into three stages to solve these problems and is easy to get
2. Our model estimates the damaged conditions and get a series of reasonable data.
3. Our counting formulas are simple and easy to edit the programs and deal with the data.

Weakness

We consider the problem incompletely,and haven't gotten the precise model.

References

[1] China: the editing organize, 1979,*collection of the papers about the meteorite shower in JiLin*, the Publishing House of Science.

[2] Russia: ∅.A.Bom,1963 ,*exploring physics*,the Publishing House of Science.

[3] Japan: 宇津徳治 ,1981,*seismology*,the Publishing House of Seismology.

[4] Australia: A.E.Stoke ,1986 ,*principle of the earth motive power*, the Publishing of Seismology.