Math 141/142 Section B
Summer 2007 Quiz 5

Name: "Answer Key"

1. Find the exact value of $\sin(2\theta)$ and $\cos(2\theta)$ if $\cos\theta = -\frac{1}{3}$ and $\frac{\pi}{2} < \theta < \pi$.

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = 2\cos^2\theta - 1$

so we need to find $\sin\theta$.

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta + \frac{1}{9} = 1 \Rightarrow \sin^2\theta = \frac{8}{9}$

$\Rightarrow \sin\theta = \frac{2\sqrt{2}}{3}$

since $\frac{\pi}{2} < \theta < \pi$

Thus, $\sin(2\theta) = 2 \cdot \frac{2\sqrt{2}}{3} \cdot \frac{-1}{3} = \boxed{\frac{-4\sqrt{2}}{9}}$

$\cos(2\theta) = 2 \cdot \frac{1}{9} - 1 = \boxed{\frac{-7}{9}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{1}{3}\right)^2 - \left(\frac{2\sqrt{2}}{3}\right)^2$

$= \frac{1}{9} - \frac{8}{9}$

$= -\frac{7}{9}$

2. Solve the equation

$$\tan\left(\frac{\theta}{2} + \frac{\pi}{3}\right) = 1 \text{ where } 0 \le \theta < 2\pi.$$

$\frac{\theta}{2} + \frac{\pi}{3} = \frac{\pi}{4} + k\pi$, k integer, since tan has period π.

$\frac{\theta}{2} = \frac{-\pi}{12} + k\pi$

$\theta = \frac{-\pi}{6} + 2k\pi$, k integer

$k=0: \quad \theta = \frac{-\pi}{6} \notin [0, 2\pi)$

$k=1: \quad \theta = \frac{11\pi}{6} \in [0, 2\pi)$

$k=2: \quad \theta = \frac{23\pi}{6} \notin [0, 2\pi)$

$\vdots$

The only solution is $\boxed{\theta = \frac{11\pi}{6}}$