Math 141/142 Section B
Summer 2007 Quiz 4

Name: "Answer Key"

1. Find the exact value of

$$\sin\overbrace{\left(\cos^{-1}\frac{1}{3}\right)}^{=\theta}.$$

Let $\theta = \cos^{-1}\frac{1}{3}$. So we'll find $\sin\theta$.

$\theta = \cos^{-1}\frac{1}{3} \Rightarrow \cos\theta = \frac{1}{3}$ where $0 \le \theta \le \pi$.

$\cos\theta$ is positive, so θ is in the 1st quadrant.

$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta + \frac{1}{9} = 1$

$\Rightarrow \sin^2\theta = \frac{8}{9} \Rightarrow \boxed{\sin\theta = \frac{2\sqrt{2}}{3}}$

2. Establish the identity

$$(1 - \cos^2\theta)(1 + \cot^2\theta) = 1.$$

$$\text{LHS} = (1 - \cos^2\theta)\cdot(1 + \cot^2\theta)$$

$$= \sin^2\theta \cdot \left(1 + \frac{\cos^2\theta}{\sin^2\theta}\right)$$

$$= \cancel{\sin^2\theta} \cdot \left(\frac{\sin^2\theta + \cos^2\theta}{\cancel{\sin^2\theta}}\right)$$

$$= \sin^2\theta + \cos^2\theta = 1 = \text{RHS}.$$