

## Formula Sheet

**Arc-Length:** For a circle of radius  $r$ , a central angle of  $\theta$  radians subtends an arc whose length is  $s = r\theta$ .

**Converting degrees to radians, radians to degrees:**

$$1 \text{ degree} = \frac{\pi}{180} \text{ radian, } 1 \text{ radian} = \frac{180}{\pi} \text{ degrees.}$$

**Area of a sector:** The area  $A$  of a sector of a circle of radius  $r$  formed by a central angle of  $\theta$  radians is  $A = \frac{1}{2}r^2\theta$ .

**Circular motion:**

$$\text{Linear speed: } v = \frac{s}{t} = \frac{\text{arc-length}}{\text{time}}$$

$$\text{Angular speed: } \omega = \frac{\theta}{t} = \frac{\text{angle swept out (in radians)}}{\text{time}}$$

$$v = r\omega.$$

**Trigonometric values of the special angles in quadrant I:**

$$\sin 0 = 0 \quad \cos 0 = 1 \quad \tan 0 = 0 \quad \cot 0 \text{ is undefined}$$

$$\sin \frac{\pi}{6} = \frac{1}{2} \quad \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \quad \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3} \quad \cot \frac{\pi}{6} = \sqrt{3}$$

$$\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \quad \tan \frac{\pi}{4} = 1 \quad \cot \frac{\pi}{4} = 1$$

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \quad \cos \frac{\pi}{3} = \frac{1}{2} \quad \tan \frac{\pi}{3} = \sqrt{3} \quad \cot \frac{\pi}{3} = \frac{\sqrt{3}}{3}$$

$$\sin \frac{\pi}{2} = 1 \quad \cos \frac{\pi}{2} = 0 \quad \tan \frac{\pi}{2} \text{ is undefined} \quad \cot \frac{\pi}{2} = 0$$

**Periodic properties of trigonometric functions:**

$$\sin(\theta + 2\pi) = \sin \theta \quad \cos(\theta + 2\pi) = \cos \theta \quad \sec(\theta + 2\pi) = \sec \theta \quad \csc(\theta + 2\pi) = \csc \theta$$

$$\tan(\theta + \pi) = \tan \theta \quad \cot(\theta + \pi) = \cot \theta$$

**Graphing sinusoidal functions:**

1. If  $\omega > 0$ , the amplitude and period of  $y = A \sin(\omega x)$  and  $y = A \cos(\omega x)$  are given by

$$\text{Amplitude} = |A|, \quad \text{Period} = T = \frac{2\pi}{\omega}.$$

In this case, we concentrate on the interval  $[0, T]$  to get a graph over one period.

2. For the graphs of  $y = A \sin(\omega x - \phi)$  or  $y = A \cos(\omega x - \phi)$ ,  $\omega > 0$ ,

$$\text{Amplitude} = |A|, \quad \text{Period} = T = \frac{2\pi}{\omega}, \quad \text{Phase shift} = \frac{\phi}{\omega}.$$

In this case, we concentrate on the interval  $\left[\frac{\phi}{\omega}, T + \frac{\phi}{\omega}\right]$  to get a graph over one period.

**Inverse trigonometric functions:**

1.  $y = \sin^{-1} x$  means  $x = \sin y$  where  $-1 \leq x \leq 1$  and  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ .

2.  $y = \cos^{-1} x$  means  $x = \cos y$  where  $-1 \leq x \leq 1$  and  $0 \leq y \leq \pi$ .

3.  $y = \tan^{-1} x$  means  $x = \tan y$  where  $-\infty < x < \infty$  and  $-\frac{\pi}{2} < y < \frac{\pi}{2}$ .

4.  $y = \cot^{-1} x$  means  $x = \cot y$  where  $-\infty < x < \infty$  and  $0 < y < \pi$ .

5.  $y = \sec^{-1} x$  means  $x = \sec y$  where  $|x| \geq 1$ , and  $0 \leq y \leq \pi$ ,  $y \neq \frac{\pi}{2}$ .

6.  $y = \csc^{-1} x$  means  $x = \csc y$  where  $|x| \geq 1$ , and  $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ ,  $y \neq 0$ .

**Trigonometric identities:**

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} \quad \csc \theta = \frac{1}{\sin \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \tan^2 \theta + 1 = \sec^2 \theta \quad \cot^2 \theta + 1 = \csc^2 \theta$$

$$\sin(-\theta) = -\sin(\theta) \quad \cos(-\theta) = \cos(\theta) \quad \tan(-\theta) = -\tan(\theta) \quad \cot(-\theta) = -\cot(\theta)$$

$$\sec(-\theta) = \sec(\theta) \quad \csc(-\theta) = -\csc(\theta)$$

**Sum and difference formulae:**

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

**Double-Angle and Half-Angle formulae:**

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}}$$

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}}$$

**Product-to-Sum formulae:**

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

**Sum-to-Product formulae:**

$$\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\sin \alpha - \sin \beta = 2 \sin \frac{\alpha - \beta}{2} \cos \frac{\alpha + \beta}{2}$$

$$\cos \alpha + \cos \beta = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$$

$$\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$$

**The Law of Sines:** For a triangle with sides  $a, b, c$ , and opposite angles  $\alpha, \beta, \gamma$ , respectively,

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}.$$

**The Law of Cosines:** For a triangle with sides  $a, b, c$ , and opposite angles  $\alpha, \beta, \gamma$ , respectively,

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta.$$

**Area of a triangle:**

$$A = \frac{1}{2}ab \sin \gamma = \frac{1}{2}bc \sin \alpha = \frac{1}{2}ac \sin \beta$$

**Heron's formula:**  $A = \sqrt{s \cdot (s - a) \cdot (s - b) \cdot (s - c)}$  where  $s = \frac{a + b + c}{2}$ .