Name:

1. **Continuous Density Function**

   The length of time (in hours) taken by a student to complete a one hour exam is a continuous random variable X with probability density function f_X defined by $f_X(x) = cx^2 + x, 0 < x \leq 1$, for some constant c, and zero otherwise.

   (a) Find the value of c.

   (b) By integration, find the distribution function of X, F_X.

   (c) Find the probability that a student completes the exam in less than half an hour.

   (d) Given that a student takes longer than fifteen minutes to complete the exam, find the probability that he/she requires at least half an hour.

   (e) In a class of sixty students, find the probability that at most three students complete the exam in fewer than fifty minutes.

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2. **Applications to the Central Limit Theorem**

   (a) If there is, on average, a crack in the road every 5 m, and number of cracks on the road can be modelled as a Poisson Process (with appropriate parameter), then what is the chance that you will find 5 cracks in a 60 m section of road? more than 525 cracks in 2.5 km (2500 m) of road?

   (b) A rookie is brought to a baseball club on the assumption (based on his minor league performance) that he has a .300 batting average (Batting average is the ratio of the number of hits to the number of times at bat). Assume that hits can be considered Bernoulli trials with probability .3 for success.

      i. Compute the probability that if he bats 300 times, his batting average turns out to be 0.283 or less (i.e, he has 85 or less hits in these 300 times).

      ii. Suppose he actually bats 300 times in his first year and his average for the first year turns out to be 0.283 (=85 hits). Based on your answer for (a) discuss, whether such a low average could be considered just bad luck or whether the rookie should be sent back to the minor leagues.

   (c) If height of an adult in a population has an unknown distribution with mean 160 and s.d 10 (in some units), and 30 people are sampled from the population, find

   (a) Distribution of $\bar{X} = \frac{1}{30}(X_1 + \ldots + X_{30})$

   (b) Distribution of $S = X_1 + \ldots + X_{30}$

   (c) Find probability that this average of 30 height observations will turn out to be more than 164.

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3. **Loading Trucks** Consider a sequence of packages. Each package is loaded independently onto either a red truck (with probability p) or a green truck (with probability $1 - p$).

   (a) Let R be the total number of items selected for the red truck out of the first n trucks. Determine the pmf, expected value and variance of the random variable R.

   (b) Let X denote the first package number that is loaded in the red truck (i.e $X = 5$ means the fifth package is the first package that is loaded in the red truck). Find the pmf, expected value and variance of the random variable X.

4. **Sum of Variables** Let X and Y be two exponential random variables with densities

$$f_X(x) = 2e^{-2x} \qquad \text{and} \qquad f_Y(y) = e^{-y}$$

   (a) Let $W = X + Y$. Determine the density function f_W.

   (b) Let $Z = Y^2$. Determine the probability that $Z > 4$.

   (c) If (X, Y) are jointly distributed as a Bivariate normal with parameters $(\mu_X = 5, \mu_Y = 6, \sigma_X = 1, \sigma_Y = 2, \varrho = -0.5)$, then find the density, f_Z of $Z = X + Y$. What is $E(Z), Var(Z), P(Z > 12)$

5. **Continuous random Variables:**

   Evaluate the following quantities for different assumptions on the random variable X given in (a), (b) and (c) below:

$$E(X),\ \ Var(X),\ \ P(X > 1),\ \ P(X < 1)$$

   (a) $X \sim Exp(\lambda = 1)$

   (b) $X \sim N(0.5, 0.5)$

   (c) $X \sim Erlang(k = 2, \lambda = 1)$