

Stat322

Counting Examples

- (a) Ninety students of a class are split into 3 sections.
- (a) If the section size is not fixed, how many different possibilities are there to form sections?
each student can join one of the 3 sections: so there are 3^{90} possible sections. Note that here sections with 0 students is a possibility
- (b) If the sections are fixed to size 30, how many different possibilities are there?
 $\binom{90}{30} \binom{60}{30} \binom{30}{30} = \binom{90}{30} \binom{60}{30}$ many possibilities
- (b) A soccer team has 11 players. Before a game starts, the players come out of the team box one after the other. How many different orders are possible?
 11!
- (c) The coach has to decide on five players when the game comes to a penalty shoot-out. How many possibilities does he have?
 $\binom{11}{5}$
- (d) The coach has to decide on five players when the game comes to a penalty shoot-out and give an order, in which they have to shoot. How many possibilities does he have?
 $\binom{11}{5} \times 11! = P(11, 5) = \frac{11!}{6!}$
- (e) Ten players form two basketball teams. How many possibilities for building different teams are there?
If the teams are of same size, then $\binom{10}{5} \binom{5}{5} = \binom{10}{5}$ many possibilities. Otherwise the solution will be similar to part (a) of problem (a) above.
- (f) Some student club consists of 5 statisticians, 10 computer scientists and 15 mathematicians. For a committee six students are elected, two from each department. How many different possibilities for the committee members do exist?
 $\binom{5}{2} \binom{10}{2} \binom{15}{2}$
- (g) Birthday problem: n people attend a party. We assume that every person has an equal probability of being born on any day during the year, independently of everybody else. What is the probability that at least two people have the same birthday? (no leap years for simplicity)
This probability = $1 - P(\text{no two people have same birthday}) = 1 - \frac{P(365, n)}{365^n}$. Look at the graph below where the probability is plotted over different group sizes (n). As you can see, for a class size as ours (around 55 people), probability of having 2 two people with same birthday is very high (close to 1)
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- (h) What is the probability that the sum of two dice is greater than 10?
out of $6^2 = 36$ possible outcomes 3 outcomes $((5,6), (6,5), (6,6))$ results in sum being (strictly) greater than 10. So the probability is $\frac{3}{36} = 1/12$
- (i) A six sided die is rolled three times independently. What is more likely: a sum of 11 or a sum of 12?
there are $6^3 = 216$ many outcomes, of which sum of "11" happens with 27 ways and "12" happens in 25 ways. Hence, a sum of 11 is more likely. Just write down all the possibilities (order matters here) and check if they add up to 11 or 12