

Stat 322 - Homework 8

Maximum score is 20 points.

1 Noisy Communication

Suppose that $X_n, n = 1, 2, 3, \dots$ is a Bernoulli process with parameter p , i.e. $P(X_n = 1) = p$ for all n .

Suppose that $W_n, n = 1, 2, 3, \dots$ is another Bernoulli process with parameter s .

We assume that the two random processes are completely independent of each other (that is, any collection of samples of X_n is independent from any collection of W_n 's). We form a new random process $Y_n, n = 1, 2, 3, \dots$ by defining

$$Y_n = X_n \oplus W_n,$$

where the \oplus operation denotes mod 2 addition. This setup can be thought of as taking an input digital signal X_n and sending it across a binary channel to a receiver. The binary channel can cause an error between the input X_n and output Y_n with probability s . Such a communication channel is called an additive noise channel because the output is the input plus an independent noise process (where plus here means mod 2).

- It is easy to see that Y_n is a Bernoulli process. Find its parameter, i.e. compute $P(Y_n = 1)$
- Write down the distribution of the number of signals sent until the k th 1 is sent. Also find the distribution of number of signals received until the k th 1 is received.
- If the signal 0 was sent, what is the probability that a 0 is received? If the signal 1 was sent, what is the probability that a 1 is received?
- Fix n . Find the conditional pmf $p_{X_n|Y_n}(k | j)$, for all values of k, j .
- Fix n . Find an expression for the probability of error, i.e. $P(Y_n \neq X_n)$.

(5 × 2=10 points)

2 Bridges in Madison County

A train bridge is constructed across a wide river. Trains arrive at the bridge according to a Poisson process of rate $\lambda = 3$ per day.

- Find probability that the first train arrives after day 1.
- Find probability that the 3rd train arrives after 1 day.
- If one train arrives on day 1, find the probability that there will be no trains on days 2, 3, and 4.
- Find the probability that no trains arrive in the first 2 days, but 4 trains arrive on the 4th day.
- Find the probability that it takes more than 2 days for the 5th train to arrive at the bridge.

(5 × 2=10 points)