Stat 322 - Homework 7

Maximum score is 20 points.

1 Apple-Tree Farm

Able and Baker are both apple-tree farmers (they grow apple trees). Assume, that apple trees grow according to a normal distribution.
On the Able Farm, trees grow with a mean of 1m per year and a standard deviation of 25 cm. Baker manages to get an average growth of 1.1m per year with a standard deviation of 35cm.
For the following questions again state each time which random variable you use and what distribution assumption you make.

(a) What is the probability for a tree on the Able Farm to grow more than 1.2m in a year? what if the tree was on the Baker Farm?

(b) What is the probability for a tree on the Able Farm to grow less than .8m in a year? what if the tree was on the Baker Farm?

(c) What is the probability for a tree on the Able Farm to grow between 0.7m and 0.9m in a year? what if the tree was on the Baker Farm?

(d) Assume, you've got two trees. One from Able and one from Baker. What can you say about the difference D in their heights? What is the distribution of D?

(e) On average, trees from the Baker farm will grow more than Able's trees. But what is the exact probability that a Baker tree has grown more than an Able tree in one year?

(1× 5=5 points)

2 Two Dimensional Density

Suppose that X and Y have a joint probability density function

$$f(x,y) = \begin{cases} k(x+2y) & \text{if } (x,y) \in [0,1] \times [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find k such that $f(x,y)$ actually IS a density function.

(b) Find the marginal density of Y.

(c) Find $E[Y]$.

(d) Find the density of $Z = Y^2$

(e) Find the conditional density $f_{X|Y}(x \mid y)$.

(f) Find $P(X \geq 0.5 \mid Y = 0.5)$.

(1+1+1+2+1+2=8 points)

3 Grand Central Station

Two friends A and B meet every morning at the Grand Central Station around 7 am. Suppose the actual times they arrive are independent and uniformly distributed between 6:55 am and 7:05am. Let Z denote the time between arrivals, i.e. $Z =$ time B arrives - time A arrives (can be negative!).

(a) What is the range of values of Z (i.e $Im(Z)$)?

(b) Find the density of Z (derive a formula similar to the *convolution formula* derived in class).

(c) Find the probability that A waits for at least 5 minutes before B arrives.

(1+4+2=7 points)