

Stat 322 - Homework 5

Maximum score is 20 points.

1 Calculators and random numbers

When you generate a random number using your calculator, the random number you get is uniformly distributed over the interval $(0, 1)$. Suppose 50 people in Stat 322 class generate one random number each (independently).

X = number of people that get a random number that is more than 0.98

- What is the distribution of X ? Name and write parameters of the distribution or write down the p.m.f
- What is the expected number of people to get a random number greater than 0.98?
- What is the probability that less than a tenth of the class (i.e 4 or less students) get a random number greater than 0.98?

(1+1+2=4 points)

2 Density Functions of continuous variables

- What are the two properties of a probability density function $f(x)$?
- Which of the following are valid probability density functions? Explain why or why not, a yes or no is not a sufficient answer.

$$f(x) = \begin{cases} x^2 + 2x, & \text{for } -1 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

$$g(x) = \begin{cases} 2x, & \text{for } 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

- Find expected value and variance of the random variable X with distribution function

$$F_X(t) = \begin{cases} 0 & \text{for } t < 0 \\ 1 - e^{-3t} & \text{for } t \geq 0 \end{cases}$$

(1+2+2=5 points)

3 more on Continuous Random Variables

For a continuous random variable X the following function is given:

$$f(x) = \begin{cases} k(2 - 2x) & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- Find k , so that $f(x)$ is a density function.
- Compute $P(X = 0.5)$.
- Compute $P(X \leq 0.5)$.
- Find $E[X]$ and $\text{Var}[X]$.

(1 + 1 + 1 + 2= 5 points)

4 Customers in a Bank

In the Ames International Campus Bank (open 24h every day) 5 customers arrive on average during an hour. Answer the following questions. First write down the problem in terms of some random variables and write down what distribution assumptions you make about these random variables.

- (a) What is the probability that during an hour no customer arrives?
- (b) What is the probability that during an hour, 7 or more customers arrive?
- (c) What is the probability that there's more than 30 minutes between the 2nd and 3rd customer of the day?
- (d) What is the probability that you have to wait less than an hour for seven customers to arrive?
- (e) How many minutes do you expect to wait until the 12th customer arrives?
- (f) How many minutes do you expect to wait on average between arrivals?

(6 points)