

Stat 322 - Homework 4

Maximum score is 20 points.

1 Finding Antony Smith

You are trying to locate an old high school friend who lives in Chicago. Unfortunately, your friend's name is Anthony Smith and the Chicago phone book lists phone numbers for 24 different people named Anthony Smith.

- If you call 10 of these Anthony Smith's at random, what is the probability that you will call your friend? (Assume that your friend's phone number is listed in the phone book, and that you don't call anybody twice.)
- Let X be the number of calls you need to make until you find your friend (note that at most you need to make 24 calls). Give the probability mass function for X .
- How many calls do you expect to make until you find your friend? (Again, assume that your friend's phone number is listed in the phone book, and that you don't call anybody twice.)

(4 points)

2 Snow in October

After October 1st the probability that a blizzard will occur on any particular day in the Midwest of Northern America is 0.1.

To simplify the problem, assume for the following questions that this probability is constant from Oct 1st onwards.

- What is the probability that there won't be a blizzard in the first 28 days of October?
- What is the probability that the first blizzard will occur on October 14?
- What is the expected date for the first blizzard (starting October 1st)?

(4 points)

3 Tossing a Coin

A fair coin is tossed 10 times. Let X be the random variable corresponding to the difference between the number of heads and the number of tails observed (i.e. $(\# \text{ of } H) - (\# \text{ of } T)$).

- Find the image of X and compute the probability mass function of X . (Hint: Instead of looking at X directly, define a new random variable Y , which just counts the number of heads in 10 tosses. What is the relationship between X and Y ?)
- Draw a diagram to show the probability mass function. Would you be able to guess the expected value of X from the diagram? Verify by computing the expected value of X .
- Compute the variance and standard deviation of X .

(4 points)

4 Deaths from Horse Kicks

After analyzing data from the Prussian cavalry for a period of 20 years, statisticians came to the conclusion that the number of deaths from horse kicks follows a Poisson distribution with $\lambda = 0.6$.

Let X be the number of deaths from horse kicks in 20 years.

What is the probability that

- a) no deaths occurred?
- b) one or two deaths occurred?
- c) 3 or more deaths occurred?
- d) more than 3 deaths occurred?

(4 points)

5 Discrete Compound PMFs

The joint pmf of two discrete r.v. X and Y is given as:

$X \backslash Y$	-1	0	1
-2	1/16	1/16	1/16
-1	1/8	1/16	1/8
1	1/8	1/16	1/8
2	1/16	1/16	1/16

- a) Find the following probabilities:
 - (a) $P(X \geq 2)$
 - (b) $P(X > Y)$
 - (c) $P(Y > 0)$
- b) Are X and Y independent?
- c) Are X and Y uncorrelated?

(4 points)