Stat 322 - Homework 3

Due date is Friday.

1 Driving Test

An individual repeatedly attempts to pass his/her driving test. Suppose that the probability that the test is passed is 0.25, and that the results of successive tests are independent. Let X be the random variable corresponding to the number of tests taken until the individual passes. Find the probability mass function of X, and evaluate the probability that

(a) the test is passed in three or less attempts,

(b) five or more attempts are necessary to pass the test.

(5 points)

2 Sum of two Dice

Assume, you throw two dice. Denote by X the random variable for their sum. Find the probability mass function of X, i.e fill out the following table:

x	2	3	4	...
$P(X = x)$				

(a) Is it more likely to get a sum of less than 4 or to get a sum of at least 9?

(b) What is the probability to get a sum of at least 11?

(c) Assume, one of the dice rolled off the table and you have to toss it again. The other die shows a 6. What is now the probability to get a sum of at least 11?

(5 points)

3 Racing Candidate

Suppose that 40% of a large population are in favor of candidate A. A pollster selects a random sample of 20 people and asks them about their opinion about the candidate. Let X be the number of people in favor of candidate A.

(a) What is the sample space of X, what is the distribution of X?

(b) What is the probability that 10 people in the sample are in favor of candidate A?

(c) What is the probability that between 6 and 10 people in the sample are in favor of candidate A? (limits are included)

(d) What is the probability that the majority of people in the sample favors candidate A.

(e) What is $E(X)$ and $Var(X)$?

(5 points)

4 Birthday on New Year's Day

The birthdays of 500 pupils in a school are recorded. Let X be the random variable corresponding to the number of these pupils who have their birthday on New Year's Day. (For simplicity: do not regard leap years.)

(a) Find the probability mass function for X. - DO NOT write down all possibilities for X, but try to find a function.

(b) The function

$$\frac{\lambda^x}{x!} \cdot e^{-\lambda},$$

with $\lambda = \frac{500}{365}$ provides an approximation to the above probability mass function (i.e, this function is an approximation for $p_X(x)$ for $x = 0, 1, \ldots, 500$). Verify the approximation numerically for $x = 0, 1, 2, 3, 4, 5, 6$.

(5 points)

5 Hat Problem

This is an extra problem. Anything above 20 you score in this HW, will go towards making your first two HW scores better. For example, if with this problem you score 23 and you have a 19 and 18.5 in the first two HWs, then with this HW, your scores for the 3 HWs will be: HW 3 : 20 (3 points left), HW 1: 20 (1 point used up, 2 points left), HW 2: 20 (1.5 points used up, 0.5 points left) - the extra 0.5 points will NOT carry over to future HWs.

Three players enter a room and a red or blue hat is placed on each person's head. The color of each hat is determined by a coin toss, with the outcome of one coin toss having no effect on the others. Each person can see the other players' hats but not his own. No communication of any sort is allowed, except for an initial strategy session before the game begins. Once they have had a chance to look at the other hats, the players must simultaneously guess the color of their own hats or pass. The group shares a hypothetical \$3 million prize if at least one player guesses correctly and no players guess incorrectly.
One obvious strategy for the players, for instance, would be for one player to guess "red" (no matter what he sees) while the other players pass.

(a) What is the expected amount of money the players win following the above strategy?

(b) Suggest a different strategy and compute the expected win for it. (1 point extra credit, if your strategy is better than the one above!)

(2+3=5 points)