GEOMETRIC OBJECTS WITH A MORE COMBINATORIAL FLAVOR
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Abstract

This research is a synergy between algebra, geometry, and combinatorics. We are studying geometric objects, defined over Finite Fields, with a view to combinational flavor and the ground for the results of the investigation of combinatorial problems in geometry and classification. We call BLT sets, being a vector space over a Finite Field, and great interest to Finite geometry and algebraic analysis across the whole body of the objects but have been explored in a long time-thousand planes, generalized quadrangles, block. On the other hand, there are objects that are invariant under a Finite group. So examples in the Coxeter groups, which are an important group of quad systems, and have classification. The geometric objects are generalizations on the roots and Coxeter groups have been as an example group of BLT sets. For example, the automorphism group of a BLT sets of the characterizations of B and N is a Coxeter group of type B and in order of B is clearly related to the group. Similar behavior is not the same with other examples. Because of geometric, we consider an example of a transitive BLT set in the n-sphere field of order 27 and present the results of two pairs of elements in the 2-dimensional projective linear groups corresponding to the generators of the groups of order 17 and 1, respectively.

BLT sets

A BLT set is a set X of q + 1 points of the generalized n-gon Q(q, n), such that no point of Q(q, n) is collinear with more than 2 points of X. BLT sets are named after Laura Bader, Eugenio Luning and Jeff Tham who first studied them in 1990 and are closely related to the group of the quadratic cone, elation generalised quadrangles and certain translation planes.

Projective Geometry

Desargues' Theorem: If the three straight lines joining the corresponding vertices of two triangles ABC and A'B'C' all meet in a point (the perspector), then the three intersections of the three straight lines joining the corresponding vertices of two triangles ABC and A'B'C' are collinear.

Why BLT sets?

BLT sets of order q would give rise to translation planes of order q^2 (that is, with q^2 + q points and the same number of lines). Thus, a BLT set of order 27 would create a really big plane. If the BLT set is linear (it arises from the linear block) then the projective plane will be the Desarguesian plane PG(q^2, 2). Thus, listing many non-linear BLT sets means listing many non-Desarguesian projective planes.

Example of a BLT set of lines

The set of q + 1 tangent lines to the twisted cubic form a BLT set in W(q, 2), for q odd.

AG(1, q) = AG(2, q)

PG(2, q) = PG(1, q)

The equation of the tangent line is:

\[ \sum_{i=1}^{n} \alpha_i x_i = 0 \]

Homogeneous polynomials:

\[ p_i = x_1 \]

\[ q_i = x_2 \]

\[ r_i = x_3 \]

Hence the tangent lines are:

\[ \{x_1 = \alpha x_2 \} \]

\[ \{x_2 = \beta x_3 \} \]

\[ \{x_3 = \gamma \} \]

Groups of Symmetry


Cartan matrices, Coxeter graphs and Dynkin diagrams

Example - consider F_4

Dynkin Diagram

Coxeter Graph

\[
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
-1 & 2 & -2 & 0 \\
0 & -1 & 0 & -1 \\
0 & 0 & 0 & 0 \\
\end{array}
\]

B-N pairs

A: B, N is a pair of subgroups B and N of a group G such that the following axioms hold:

1. G is generated by B and N.
2. The intersection B and N is a normal subgroup of N.
3. The group W = N/B is generated by a set of elements \( w_i \) of order 2.2, for i in some non-empty set I.
4. If \( w_i \) is one of the generators of W and \( w \) is any element of W, then \( wBw^{-1} \) is contained in the union of \( BwB^{-1} \) and \( Bw^{-1}B \).
5. No generator \( w \) normalizes B.

For our convenience, B is the stabilizer of a maximal flag and N represents the monomial matrices. Hence B represents the monomial matrices that stabilize the maximal flag.

Results for D_4

Future Directions

1. \([G/N, G/N] = \{0\} \) and \([D_4/N] = \{0\} \), so we could construct the corresponding BN pair using the stabilizer chain of fundamental roots.
2. Investigate more BLT groups, some generalize and some don’t.
3. Find an algebraic equation for the points, by a finding low degree homogeneous polynomial whose zero set is the points. The set needs to be partitioned and maybe low degree polynomials exist for each of the parts of the partition. This would mean the set is the intersection of varieties.

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Bibliography