1. (20 points) (i) Compute the following limits. Give your reasons if the limit does not exist.

a) \[ \lim_{(x,y) \to (1,-1)} \frac{x^2 + 3xy + 2y^2}{x^2 + 2y^2} \]

b) \[ \lim_{(x,y) \to (0,0)} \frac{2x + y}{\sqrt{x^2 + y^2}} \]

(ii) Let \( w = x^2y + xy^2 + 2y \), where \( x = s(1 + \cos t) \) and \( y = s(1 + \sin t) \). Here \( s \) and \( t \) are parameters. Compute \( \frac{\partial w}{\partial s} \) when \( s = 1 \) and \( t = \frac{\pi}{2} \).
2. (30 points) Let \( f(x, y) = x^2 - 3xy + 3y^2 + 3y \).

(i) Find the equation of the tangent plane to the surface \( z = f(x, y) \) at the point \((2, 1)\). Simplify your answer and write it in the form \( ax + by + cz = d \).

(ii) Find all points \((x, y)\) at which the tangent plane to the surface of \( z = f(x, y) \) is parallel to the plane \( 10x - 6y - 2z = 5 \). Here \( f(x, y) = x^2 - 3xy + 3y^2 + 3y \) is the same function as in part (i).

(iii) Compute \( \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \) and simplify your answer. Here \( f(x, y) = x^2 - 3xy + 3y^2 + 3y \) is the same function as in part (i).
3. (30 points) Let $f(x, y)$ be a function differentiable at $(1, 2)$. It is given that 
$f(1, 2) = 4, \ \frac{\partial f}{\partial y}(1, 2) = -5$ and the directional derivative at the point $(1, 2)$ in the
direction of the vector $3i + 4j$ is equal to 8.

(i) Compute $\frac{\partial f}{\partial x}(1, 2)$.

(ii) Find a unit vector along the direction of maximum rate of change of $f$ at the point
$(1, 2)$. Also compute the maximum rate of change.

(iii) Find a unit vector $u$ so that the directional derivative $D_uf(1, 2) = 0$. 
4. (20 points) Let \( f(x, y) = x^2 - 2xy + \frac{1}{3}y^3 - 3y + 7 \).

(i) Compute all the critical points.

(ii) Classify all the local maxima, local minima and the saddle points.