Chapter 13 problems from old finals

Spring 2012 - Administered

5. Evaluate
\[ \int_S xy \, dA, \]
where \( S \) is the region in the first quadrant inside \( x^2 + y^2 = 9 \) and outside \( x^2 + y^2 = 4 \).

6. Find
\[ \int_0^1 \int_{x-x^2}^{2-x^2} 6x \cos \left((x^2 + y)^3\right) \, dy \, dx \]
by making the substitutions \( u = x^2 + y \) and \( v = x \).

Spring 2012 - Alternate

5. Rewrite the iterated integral
\[ \int_0^1 \int_0^{1-x} \int_0^{3-y} z \sqrt{4 - x^2 - y^2} \, dz \, dy \, dx \]
as an iterated integral with order of integration \( dx \, dz \, dy \) and evaluate the integral.

6. Let \( T \) be the solid which consists of all the points satisfying
\[ 0 \leq x^2 + y^2 \leq 4 \quad \text{and} \quad 0 \leq z \leq 3 + \sqrt{4 - x^2 - y^2}. \]
(This solid is a cylinder of radius 2 and height 3 topped off with a hemisphere.) Given that for this solid \( \delta(x, y, z) = 1 \) find the center of mass. (You can use symmetry and volumes of solids to simplify the calculations.)

Fall 2011 - Administered

5. A lamina with density \( \delta(r, \theta) = 1 \) has the shape of a cardioid \( r = 1 + \cos \theta \). Set up integrals for the mass of the lamina, and its moments with respect to the x-axis and y-axis. (It is not required to evaluate any of these integrals.) Symmetry guarantees that one of the moments is zero. Which one?

6. Set up an integral to find the volume of the solid that the cylinder \( r = \cos \theta \) cuts out of the sphere of radius 1 centered at the origin. (It is not required to evaluate the integral.)

Fall 2011 - Original

4. Let \( R \) be the region in the first quadrant bounded by the coordinate axes and the ellipse \( 4x^2 + y^2 = 4 \). Evaluate the double integral
\[ \int_R (x + 2y) \, dA. \]

7. Use the change of variables \( u = x + 2y \) and \( v = x - 2y \) to evaluate \( \int_S (3x + 6y)^2 \, dA \) where \( S \) is a parallelogram with vertices \((2, 0), (0, 1), (-2, 0), (0, -1)\).

Spring 2010

5. Find the center of mass of the solid inside the sphere \( \rho = 2 \), below the cone \( \phi = \pi/3 \) and above the plane \( z = 0 \), if the density is proportional to the distance from the origin. Use symmetry where possible.

7. Evaluate
\[ \int_R (x + y) \sin(x - y) \, dA, \]
where \( R \) is a triangle with vertices \((0, 0), (\pi, \pi)\) and \((\pi, -\pi)\).

Spring 2009

5. Find the mass and the center of mass of a thin plate with density function \( \delta(x, y) = y \) occupying the triangle with vertices \((0, 0), (1, 1), \) and \((-1, 1)\).

6. Convert the integral below from cylindrical coordinates to an equivalent integral in a) Cartesian, b) spherical coordinates. DO NOT EVALUATE.
\[ I = \int_0^\pi \int_0^r \int_0^{\sqrt{3r}} r^2 \sin \theta \, dz \, dr \, d\theta. \]