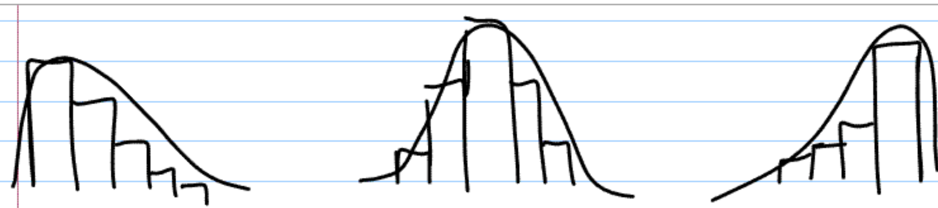
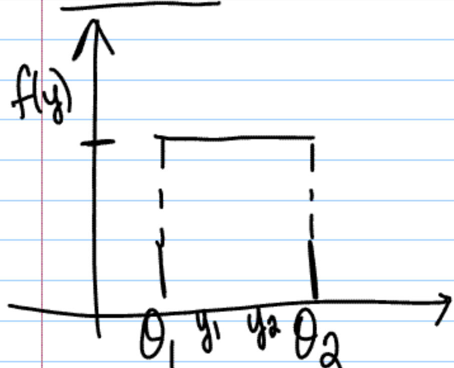
Uniform Distribution

Note Title 10/29/2008



Uniform



θ_1 & θ_2 are parameters

$$f(y) = \begin{cases} \dfrac{1}{\theta_2 - \theta_1} & \theta_1 \le y \le \theta_2 \\ 0 & \text{elsewhere} \end{cases}$$

$$F(y) = \int_{-\infty}^{y} f(t)\,dt$$

$$= \cancel{\int_{-\infty}^{\theta_1} 0 \, dt} + \int_{\theta_1}^{y} \frac{1}{\theta_2-\theta_1} dt = \frac{y-\theta_1}{\theta_2-\theta_1}$$

Graphs – $\theta_1 = 0$, $\theta_2 = 1$ (Standard Uniform Dist).

μ = midpoint of interval $[\theta_1, \theta_2]$

$$= \frac{\theta_1+\theta_2}{2}$$

$$\sigma^2 = E(Y^2) - (E(Y))^2 = \frac{\theta_2^3 - \theta_1^3}{3(\theta_2-\theta_1)} - \left(\frac{\theta_1+\theta_2}{2}\right)^2$$

$$E(Y^2) = \int_{\theta_1}^{\theta_2} y^2 \frac{1}{\theta_2-\theta_1} dy = y^3 \Big|_{\theta_1}^{\theta_2} \cdot \frac{1}{3(\theta_2-\theta_1)}$$

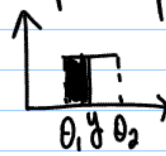
$$\frac{\theta_2^3-\theta_1^3}{3(\theta_2-\theta_1)}$$

messy algebra

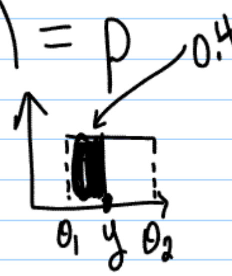
$$\boxed{\sigma^2 = \frac{(\theta_2-\theta_1)^2}{12}}$$

Uniform – unif

- dunif – density values (nice plots, but no help for prob)
- punif – $F(y) = P(Y \le y)$



- qunif – y such that $P(Y \le y) = p$

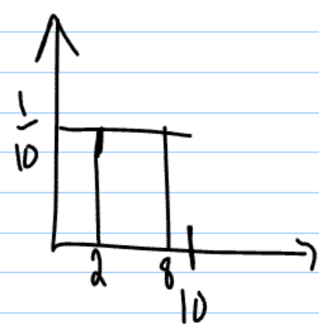


- runif - generate uniform values

1. (a) $\theta_1 = 0$ & $\theta_2 = 10$

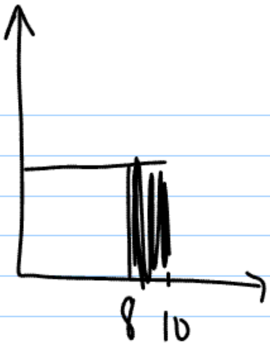
(b) $f(y) = \begin{cases} \frac{1}{10} & 0 \le y \le 10 \\ 0 & \text{elsewhere} \end{cases}$

(c)



$(8-2) \cdot \frac{1}{10} = 0.6$

(d)



$(10-8)\cdot\frac{1}{10}$

$P(Y\geq 8) = 1 - P(Y\leq 8) = 0.2$

(e) $E(Y) = 5$ $\quad V(Y) = \frac{10^2}{12} = \frac{100}{12}$

(2.) Y is uniform $\theta_1 = 0$ + $\theta_2 = 1$ st. uniform

$W = a + (b-a)Y \quad a<b \qquad a=2$

$b=5$

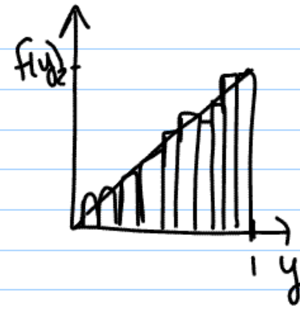
$F_W(w) = \frac{w-a}{b-a}$ is uniform $\theta_1 = a,\ \theta_2 = b$

$F_W(w) = P(W\leq w) = P(a+(b-a)Y\leq w)$

$= P\left(Y\leq \frac{w-a}{b-a}\right) = \frac{w-a}{b-a}$

Generating Data

$$f(y) = \begin{cases} 2y & 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$



① $P(Y \le y) = F(y)$

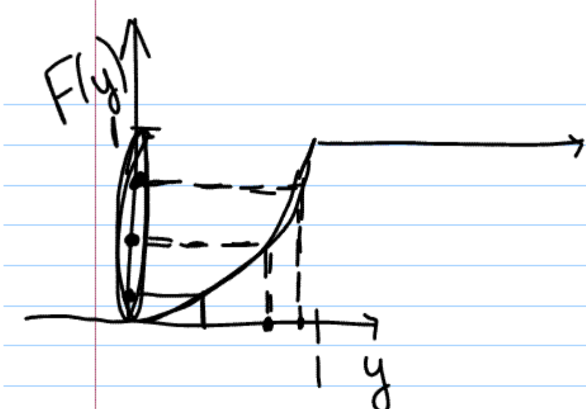
$$F(y) = \begin{cases} 0 & y < 0 \\ y^2 & 0 \le y \le 1 \\ 1 & y > 1 \end{cases}$$

for $0 \le y \le 1$

$$F(y) = \int_{-\infty}^{y} f(t)\,dt$$

$$= \int_{-\infty}^{0} 0\,dt + \int_{0}^{y} 2t\,dt = t^2 \Big|_0^y$$

$$= y^2$$



Uniform Dist $\theta_1 = 0, \theta_1 = 1$; generate 10,000 values from Uniform(0,1)

F(y) is the uniform dist. values (True for all Y)

Find y so that F(y) = uniform value

$$F(y) = \sqrt{\text{uniform value}} = \sqrt{y^2}$$

$$y = \sqrt{\text{uniform value}}$$

$$f(y) = \frac{3}{8} y^2 \qquad 0 \le y \le 2$$

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{8} y^3 & 0 \le y \le 2 \\ 1 & y > 2 \end{cases}$$

Generate $F(y)$ values $\boxed{(u \text{ from } U(0,1))}$

$u = F(y)$ what is y?

$$u = \frac{1}{8} y^3$$

$$8u = y^3$$

$$2 \cdot u^{1/3} = y$$