Uniform Distribution - Section 4.4

The uniform distribution is used to model continuous data when the probability histogram is approximately a horizontal line throughout the range of the data. The uniform distribution has the following properties:

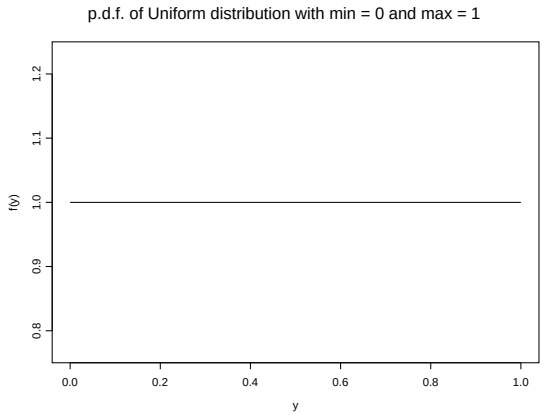
- The parameters of the uniform distribution are the minimum (θ_1) and maximum (θ_2) value. The standard uniform distribution has minimim $\theta_1 = 0$ and maximum $\theta_2 = 1$.
- The probability density function for the uniform distribution is

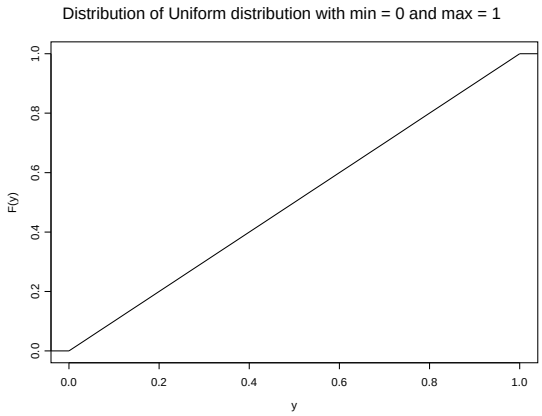
$$f(y) = \frac{1}{\theta_2 - \theta_1} \qquad \theta_1 \leq y \leq \theta_2$$

- The distribution function for the uniform distribution is

$$F(y) = P(Y \leq y) = \begin{cases} 0 & y < \theta_1 \\ \frac{y-\theta_1}{\theta_2-\theta_1} & \theta_1 \leq y \leq \theta_2 \\ 1 & y > \theta_2 \end{cases}$$

  Here are graphs of the probability density function and the distribution function of a Uniform distribution with $\theta_1 = 0$ and $\theta_2 = 1$.





- The theoretical mean of the uniform distribution is

$$\mu = E(Y) = \frac{\theta_1 + \theta_2}{2}$$

- The variance of the uniform distribution is

$$\sigma^2 = V(Y) = \frac{(\theta_2 - \theta_1)^2}{12}$$

Working with uniform random variables in R.

To find the probability $P(Y \leq y)$, the command in R is

`punif(y,a,b)`

To find the value of y so that $P(Y \leq y) = p$, the command in R is

`qunif(p,a,b)`

To generate observed values from a uniform distribution, the command in R is

`runif(numobs,a,b)`

where numobs is the number of observed values you would like to generate.

Problems.

1. Customers arrive randomly at a bank teller's window. Given that one customer arrived during a particular 10-minute period, let Y equal the time within the 10 minutes that the customer arrived. If Y follows a uniform distribution, find

   (a) The minimum and maximum values for Y.
   (b) The probability density function for Y.
   (c) $P(2 \leq Y \leq 8)$.
   (d) $P(Y \geq 8)$.
   (e) The mean and variance of Y.

2. Let Y be a uniform random variable with minimum $\theta_1 = 0$ and maximum $\theta_2 = 1$. Define $W = a + (b - a)Y$ where $a < b$.

   (a) Find the distribution function of W.
   (b) What distribution does the random variable W have?

3. Let Y be a continuous random variable with the following probability density function

$$f(y) = 2y \qquad 0 \leq y \leq 1$$

   Use R to generate 10,000 observations of the random variable Y.

4. Let Y be a continuous random variable with the following probability density function

$$f(y) = \frac{3}{8}y^2 \qquad 0 \leq y \leq 2$$

   Use R to generate 10,000 observations of the random variable Y.