Special Gamma Distributions - Section 4.6

There are two special distributions in the family of gamma distributions. They are the exponential distribution and the chi-square distribution.

1. Exponential distribution

   An exponential distribution is a gamma distribution with $\alpha = 1$. The only parameter for an exponential distribution is the scale parameter β. Exponential distributions are often used to model the length of life of electronic components.

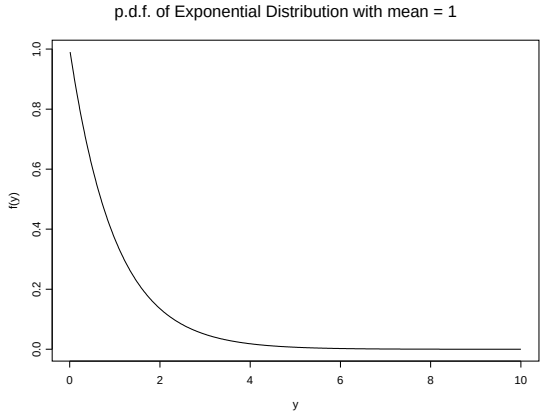
   The probability density function of an exponential distribution is

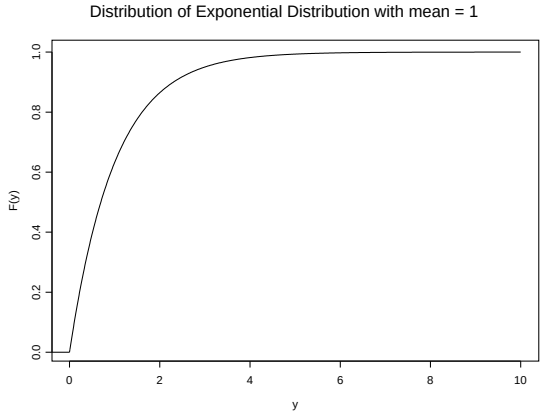
$$f(y) = \frac{1}{\beta} e^{-y/\beta} \qquad 0 \leq y < \infty$$

   The distribution function for the exponential distribution has a closed form solution. The distribution function is

$$F(y) = P(Y \leq y) = \begin{cases} 0 & y < 0 \\ 1 - e^{-y/\beta} & 0 \leq y < \infty \end{cases}$$

   Here are graphs of the probability density function and the distribution function of an Exponential distribution with $\beta = 1$.





   The theoretical mean of an exponential distribution is $\mu = E(Y) = \beta$ and the variance is $\sigma^2 = V(Y) = \beta^2$.

   Working with the exponential distribution in R.

   To find the probability $P(Y \leq y)$ the command in R is

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pexp(y,1/beta)
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   To find the value of y so that $P(Y \leq y) = p$ the command in R is

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qexp(p,1/beta)
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To generate observations from the exponential distribution the command in R is

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rexp(numobs,1/beta)
```

where numobs is the number of observations you wish to generate.

2. Chi-Square Distribution.

   A chi-square distribution is a gamma distribution with $\alpha = \nu/2$ and $\beta = 2$. The only parameter of the chi-square distribution is ν. This parameter is referred to as the degrees of freedom of the chi-square distribution.

   The chi-square distribution occurs frequently in statistical theory so we will be discussing this distribution much more in Statistics 342 next semester.

   Working with the chi-square distribution in R.

   To find the probability $P(Y \leq y)$ the command in R is

   ```
   pchisq(y,nu)
   ```

   To find the value of y so that $P(Y \leq y) = p$ the command in R is

   ```
   qchisq(p,nu)
   ```

   To generate observations from a chi-square distribution the command in R is

   ```
   rchisq(numobs,nu)
   ```

   where numobs is the number of observations you wish to generate.

Problems.

1. A manufacturing plant uses a specific bulk product. The amount of product used in one day can be modeled by an exponential distribution with $\beta = 4$ (measurements in tons).
   (a) Find the probability that the plant will use more than 4 tons on a given day.
   (b) How much of this bulk product should be stocked so that the plant's chances of running out of the product is only 0.05?

2. One-hour carbon monoxide concentrations in air samples from a large city have an approximately exponential distribution with mean 3.6 parts per million.
   (a) Find the probability that the carbon monoxide concentration exceeds 9 parts per million during a randomly selected 1-hour period.
   (b) A traffic-control strategy rediced the mean to 2.5 parts per miollion. Now find the probability that the concentration exceeds 9 parts per million.

3. Explosive devices used in mining operations produce nearly circular craters when detonated. The radii of these craters are exponentially distributed with mean 10 feet. Find the mean and variance of the areas produced by these explosive devices.