

Probability Examples - Conditional Probabilities and Bayes Theorem

1. Diseases I and II are prevalent among people in a certain population. It is assumed that 10% of the population will contract disease I sometime during their lifetime, 15% will contract disease II eventually, and 3% will contract both diseases.
 - (a) Find the probability that a randomly chosen person from this population will contract at least one disease.
 - (b) Find the conditional probability that a randomly chosen person from this population will contract both diseases, given that he or she has contracted at least one disease.
2. Of the items produced daily by a factory, 40% come from Line I and 60% come from Line II. Line I has a defect rate of 8%, whereas Line II has a defect rate of 10%. If an item is chosen at random from the day's production, find the probability that it will not be defective.
3. The completion of a construction job may be delayed because of a strike. The probabilities are 0.60 that there will be a strike, 0.85 that the construction will be finished on time if there is no strike, and 0.35 that the construction will be finished on time if there is a strike. If the job was finished on time, what is the probability that a strike occurred?
4. In a certain factory, machines I, II and III all produce springs of the same length. It is known that machines I, II and III produce defective springs 2%, 1% and 3% of the time, respectively and that machines I, II and III produce 35%, 25%, 40% of the total spring manufactured. Given the spring is defective, find the probability that the spring was produced by machine III.
5. Suppose 1% of the population of the United States is known to have HIV. For a certain blood test, the probability of testing positive for HIV when you have HIV is 99%, and the probability of testing positive for HIV when you do not have HIV is 0.1%. What is the probability that someone who tested positive for the HIV virus actually has HIV?
6. In California, 25% of all cars emit excessive amounts of pollutants. If the probability is 0.99 that a car emitting excessive amounts of pollutants will fail the state's vehicular emission test, and the probability is 0.17 that a car not emitting excessive amounts of pollutants will nevertheless fail the test, what is the probability that a car fails the test?