

Normal Distribution - Section 4.5

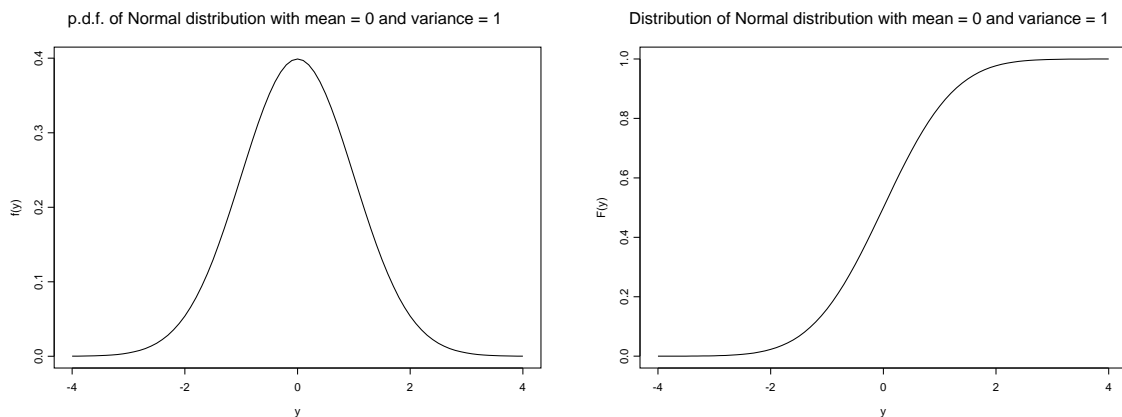
A normal distribution is used to model continuous data when the probability histogram has an approximate bell-shape. The normal distribution has the following properties:

- The parameters for the normal distribution are the mean μ and the variance σ^2 . The standard normal distribution has mean $\mu = 0$ and variance $\sigma^2 = 1$.
- The probability density function for the normal distribution is

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}} \quad -\infty < y < \infty$$

- The distribution function $F(y)$ for the normal distribution does not have a closed form solution. You must use tables or a computer package to find probabilities associated with the normal distribution.

Here are graphs of the probability density function and the distribution function of a Normal distribution with $\mu = 0$ and $\sigma^2 = 1$.



- The theoretical mean of the normal distribution is

$$\mu = E(Y) = \mu$$

- The variance of the normal distribution is

$$\sigma^2 = V(Y) = \sigma^2$$

- The normal distribution is very important in statistical theory and we will be learning much more about this distribution in Statistics 342.

Working with normal random variables in R.

To find the probability $P(Y \leq y)$ the command in R is

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pnorm(y,mu,sigma)
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To find the value of y so that $P(Y \leq y) = p$ the command in R is

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qnorm(p,mu,sigma)
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To generate observed values from a normal distribution the command in R is

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rnorm(numobs,mu,sigma)
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where numobs is the number of observed values you would like to generate.

Problems.

1. Scores on a particular achievement test are known to have a normal distribution with mean $\mu = 75$ and variance $\sigma^2 = 100$.
 - (a) Find the probability a randomly selected student will score between 80 and 90 on this achievement test.
 - (b) 30% of all students taking this achievement test will score better than what value?
 - (c) Out of 5 randomly selected students taking this achievement test, find the probability that all 5 students will score between 80 and 90.
 - (d) Out of 5 randomly selected students taking this achievement test, find the probability that 4 out of the 5 students will score between 80 and 90.

2. The 'fill' problem is important in many industries, like those making cereal, toothpaste, beer, and so on. If such an industry claims it is selling 12 ounces of its product in a container, it must have a mean greater than 12 ounces or else the FDA will crack down on the industry. However, the industry is allowed to have a very small percentage of the containers less than 12 ounces.
 - (a) If the contents Y of a container have a normal distribution with mean $\mu = 12.1$ ounces and a variance σ^2 , find σ^2 so that $P(Y < 12) = 0.01$.
 - (b) If $\sigma = 0.05$, find μ so that $P(Y < 12) = 0.01$.

3. Assume that the fill Y of a filling machine for a beverage has a normal distribution with $\mu = 12.2$ and $\sigma = 0.1$, measured in fluid ounces.
 - (a) Find $P(Y < 12)$.
 - (b) 50 bottles of this beverage are selected independently. What is the probability that at least one is under 12 ounces?