Continuous r.V.

Note Title 10/20/2008

Sample Space S

Flip a coin 3 times

Y = # of heads obtained

HHH	HHT	HTH	HTT	THH	THT	TTH	TTT
3	2	2	1	2	1	1	0

y	0	1	2	3
p(y)	1/8	3/8	3/8	1/8

finite # of simple events in S.

Y has a finite number of values

Y = # of flips needed to obtain 3 heads

negative binomial with $r=3$, $p=\frac{1}{2}$

Y has values from $\boxed{r, r+1, r+2,}$ ...

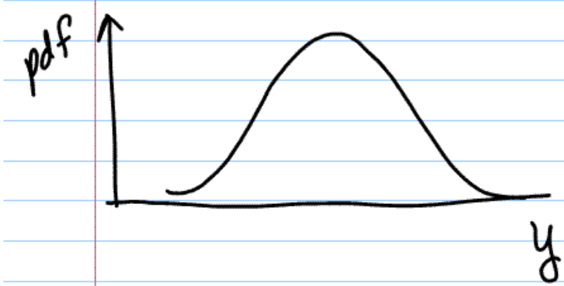
Countably Infinite number of values

Discrete random variables = r.v. only has either a finite # of values or countably infinite number of values.

Y = height of a randomly selected person.

interval of the real-line.

model as a continuous random variable.



Distribution Function

$P(Y \le y) = F(y) \qquad -\infty < y < \infty$

y	0	1	2	3
$p(y)$	$1/8$	$3/8$	$3/8$	$1/8$

$y < 0 \quad P(Y \le y) = 0 \qquad y = 1 \quad P(Y \le 1) = \frac{4}{8}$

$y = 0 \quad P(Y \le 0) = 1/8 \qquad 1 \le y < 2 \quad P(Y \le y) = \frac{4}{8}$

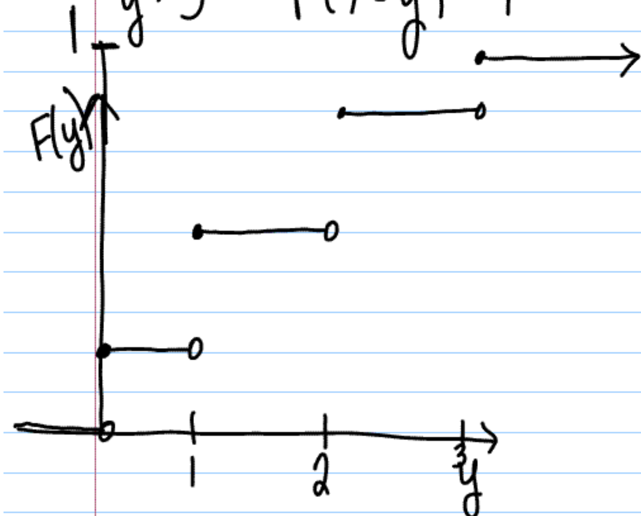
$0 \le y < 1 \quad P(Y \le y) = 1/8 \qquad y = 2 \quad P(Y \le 2) = \frac{7}{8}$

$2 \le y < 3 \quad P(Y \le y) = \frac{7}{8}$

$y = 3 \quad P(Y \le 3) = 1$

$y > 3 \quad P(Y \le y) = 1$

$$F(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{8} & 0 \le y < 1 \\ \frac{4}{8} & 1 \le y < 2 \\ \frac{7}{8} & 2 \le y < 3 \\ 1 & y \ge 3 \end{cases}$$



For discrete r.v. $F(y)$ is a step function

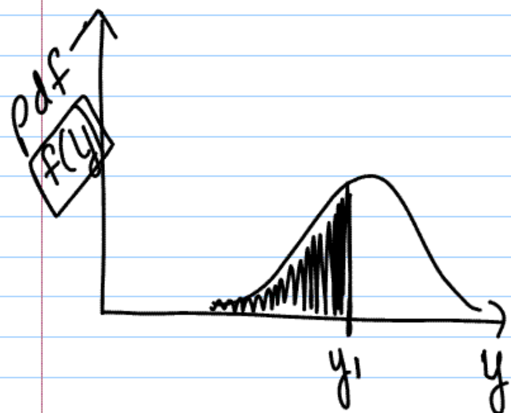
$0 \le F(y) \le 1 \qquad \lim_{y \to -\infty} F(y) = 0$

$$\lim_{y \to \infty} F(y) = 1$$

$F(y)$ is a non-decreasing function.

For continuous r.v. Y

$$F(y) = P(Y \le y) = \int_{-\infty}^{y} f(t)\, dt$$

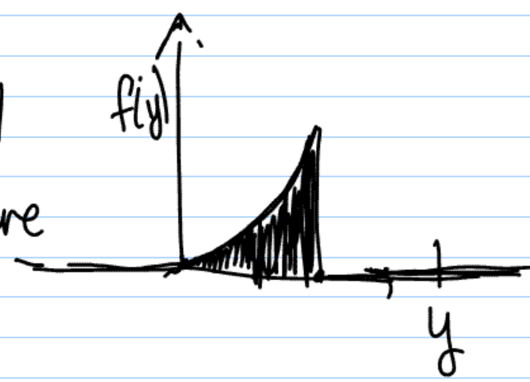


$$P(Y \le y_1) = \int_{-\infty}^{y} f(t)\, dt$$

For continuous r.v. Y $F(y)$ is continuous for all y $-\infty < y < \infty$

Y has pdf

$$f(y) = \begin{cases} 3y^2 & 0 \le y \le 1 \\ 0 & \text{elsewhere} \end{cases}$$



$F(y)$?

$y<0 \quad F(y) = \int_{-\infty}^{y} 0\,dt = 0$

$y \geq 0$, $y \leq 1$ $\quad F(y) = \int_{-\infty}^{y} f(t)\,dt = \int_{-\infty}^{0} 0\,dt + \int_{0}^{y} 3t^2\,dt$

$= 0 + t^3 \Big|_0^y = y^3$

$y>1 \quad F(y) = \int_{-\infty}^{y} f(t)\,dt = \int_{-\infty}^{0} 0\,dt + \int_{0}^{1} 3t^2\,dt + \int_{1}^{y} 0\,dt$

$= 0 + t^3 \Big|_0^1 + 0 = (1^3 - 0^3) = 1$

everywhere continuous

$$\boxed{F(y)} = \begin{cases} 0 & y < 0 \\ y^3 & 0 \le y \le 1 \\ 1 & y > 1 \end{cases}$$

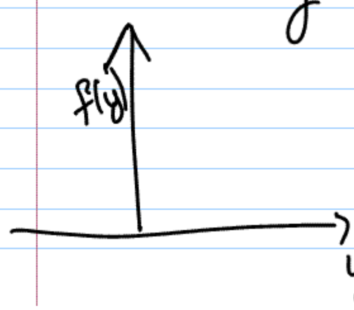
$$F(y) = \boxed{\int_{-\infty}^{y}} f(t)\,dt$$

$F(y)$ and $f(y)$

pdf prob. density function

$0 \le F(y) \le 1$ $\qquad$ $F'(y) = f(y)$

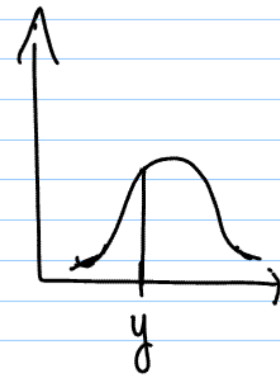
non-decreasing $\qquad$ $f(y)$ is always ≥ 0.



$$\int_{-\infty}^{\infty} f(t)\,dt = 1$$

$$\boxed{P(Y<y) = P(Y\le y)}$$

$$\int_{-\infty}^{y} f(t)\,dt = \int_{-\infty}^{y} f(t)\,dt$$

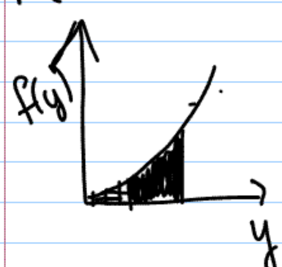


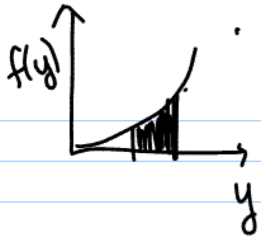
$$f(y) = \begin{cases} 3y^2 & 0\le y\le 1 \end{cases} \qquad F(y) = \begin{cases} 0 & y<0 \\ y^3 & 0\le y\le 1 \\ 1 & y>1 \end{cases}$$

$$P(Y\le 0.25) = F(0.25) = (0.25)^3$$

$$P(0.25\le Y\le 0.75) = F(0.75) - F(0.25)$$

$$= 0.75^3 - 0.25^3$$





$$\int_{0.25}^{0.75} 3y^2\,dy = y^3\Big|_{0.25}^{0.75}$$

$$= 0.75^3 - 0.25^3$$

$$P(0.25 < Y < 0.75) = P(0.25 \le Y \le 0.75)$$

Ex. 2 $\quad f(y) = \begin{cases} cy^2 & 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$

What is c?

$$\int_{-\infty}^{\infty} f(y)\,dy = 1$$

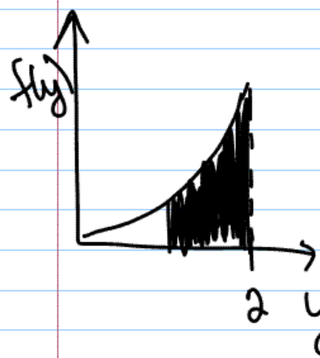
$$\int_{0}^{2} cy^2\,dy = 1$$

$$\int_{0}^{2} cy^2\,dy = \frac{c}{3}y^3\Big|_0^2 = \frac{c}{3}(8-0) = \frac{8c}{3} = 1$$

$c = \frac{3}{8}$

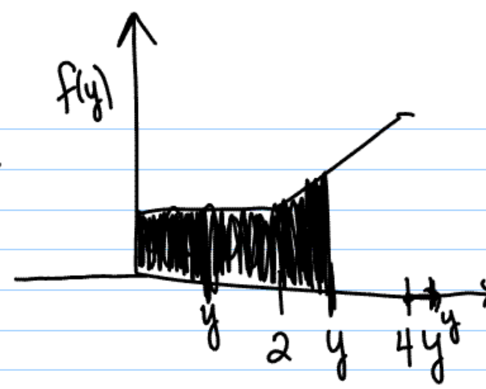
$$P(1 \le Y \le 2) = \int_1^2 \frac{3}{8} y^2 \, dy = \frac{1}{8} y^3 \Big|_1^2$$

$$= \frac{1}{8}(8-1) = \frac{7}{8}$$



Ex.3

$$f(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{8} & 0 \le y \le 2 \\ \frac{y}{8} & 2 \le y \le 4 \\ 0 & y > 4 \end{cases}$$



F(y)?

$$F(y) = \begin{cases} 0 & y < 0 \\ y/8 & 0 \le y \le 2 \\ y^2/16 & 2 \le y \le 4 \\ 1 & y > 4 \end{cases}$$

$$F(y) = \int_{-\infty}^{y} f(t)\,dt$$

$$0 \le y \le 2 = \int_0^y \frac{1}{8}\,dt = \frac{t}{8}\Big|_0^y = \frac{y}{8}$$

$$2 \le y \le 4 = \int_0^y f(t)\,dt = \int_0^2 \frac{1}{8}\,dt + \int_2^y \frac{t}{8}\,dt$$

$$= \frac{t}{8}\Big|_0^2 + \frac{t^2}{16}\Big|_2^y = \left(\cancel{\frac{2}{8}} - 0\right) + \left(\frac{y^2}{16} \cancel{- \frac{1}{4}}\right) = \frac{y^2}{16}$$