

Statistics 341

Fall 2008 - Homework Assignment #8

We will go over some of these problems during the last week of classes

1. Let Y_1 and Y_2 have the joint probability density function given by

$$f(y_1, y_2) = \begin{cases} 6(1 - y_2) & 0 \leq y_1 \leq y_2 \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Find $P(Y_1 \leq 3/4, Y_2 \geq 1/2)$.
 - (b) Find the marginal distributions for both Y_1 and Y_2 .
 - (c) Find the conditional density function of Y_1 when $Y_2 = y_2$.
 - (d) Find the conditional density function of Y_2 when $Y_1 = y_1$.
 - (e) Find $P(Y_2 \geq 3/4 | Y_1 = 1/2)$.
 - (f) Are Y_1 and Y_2 independent random variables? Explain your answer.
 - (g) Find the covariance between Y_1 and Y_2 .
 - (h) Find the correlation between Y_1 and Y_2 .
2. An environmental engineer measures the amount (by weight) of particulate pollution in air samples of a certain volume collected over two smokestacks at a coal-operated power plant. One of the stacks is equipped with a cleaning device. Let Y_1 denote the amount of pollutant per sample collected above the stack that has no cleaning device and let Y_2 denote the amount of pollutant per sample collected above the stack that is equipped with the cleaning device. Suppose that the relative frequency behavior of Y_1 and Y_2 can be modeled by

$$f(y_1, y_2) = \begin{cases} 1 & 0 \leq y_1 \leq 2, 0 \leq y_2 \leq 1, 2y_2 \leq y_1 \\ 0 & \text{elsewhere} \end{cases}$$

- (a) Find $P(Y_1 \geq 3Y_2)$. This is the probability the cleaning device reduces the amount of pollutant by one-third or more.
- (b) Find the marginal distributions for both Y_1 and Y_2 .
- (c) Find the conditional density function of Y_1 when $Y_2 = y_2$.
- (d) Find the conditional density function of Y_2 when $Y_1 = y_1$.
- (e) Find the probability that the amount of pollutant in a sample taken above the stack without the cleaner (Y_1) is 1.5 or greater given the amount of pollutant in a sample taken above the stack with the cleaner (Y_2) is 0.5.
- (f) Are Y_1 and Y_2 independent random variables? Explain your answer.
- (g) Find the covariance between Y_1 and Y_2 .
- (h) Find the correlation between Y_1 and Y_2 .
- (i) Find the expected value and variance of $Y_1 - Y_2$, the amount of pollutant removed by the cleaner.

3. Let Y_1, Y_2, \dots, Y_n be independent Poisson random variables with means λ_i for $i = 1, 2, \dots, n$. Define the mean of these n random variables to be

$$\bar{Y} = \frac{\sum_{i=1}^n Y_i}{n}$$

Find the expected value and variance of \bar{Y} .