

Statistics 341

Homework Assignment #6

Due Friday, November 7, 2008

1. Let the random variable Y have a Uniform distribution with $\theta_1 = 0$ and $\theta_2 = 1$. Show that

$$P(a \leq Y \leq a + b) \text{ for } a \geq 0, b \geq 0, a + b \leq 1$$

depends only on the value of b .

2. The cycle time for trucks hauling concrete to a highway construction site is uniformly distributed over the interval from 50 to 70 minutes.

- (a) What is the probability the cycle time will be greater than 65 minutes?
- (b) Find the mean and variance of the cycle time.

3. As a measure of intelligence, mice are timed when going through a maze to reach a reward of food. The time (in seconds) required for any mouse is a random variable Y with a density function given by

$$f(y) = \begin{cases} \frac{60}{y^2} & y \geq 60 \\ 0 & \text{elsewhere} \end{cases}$$

where 60 seconds is the minimum possible time needed to traverse the maze.

- (a) Find the distribution function $F(y)$ for the random variable Y .
 - (b) Use R to generate 10,000 observations of the random variable Y . Find the minimum and maximum values for these 10,000 observations. What do you notice about your values?
 - (c) Make a probability histogram for your 10,000 observations. Print this out and include it with your assignment. How would you describe this histogram?
 - (d) Find the mean and variance of these 10,000 observations. How do these observed values compare to the theoretical values of the mean and variance from Homework Assignment #5? Explain your answer.
4. Wires manufactured for use in a certain computer system are specified to have resistances between 0.12 and 0.14 ohms. The actual measured resistances of the wires produced by Company A have a normal distribution with a mean of 0.13 ohm and a standard deviation of 0.005 ohm.
 - (a) What is the probability that a randomly selected wire from Company A's production will meet the specifications?
 - (b) If four such wires are used in the system and all are from Company A, what is the probability that all four will meet the specifications?
 5. A soft drink machine can be regulated so that it discharges an average of μ ounces per cup. If the ounces of fill are normally distributed with standard deviation equal to 0.3 ounce, give the setting for μ so that 8-ounce cups will overflow only 1% of the time.

6. Many variables that measure physical characteristics, such as height, are considered to be normally distributed. For the past several semesters, students enrolled in Statistics 101 have filled out a questionnaire on the web. One of the questions asks for their height in inches. On the course web page, find and click on the link for the Stat 101 Female Height Data. Then copy and paste the data into R using the `scan` command. Use these data to answer the following questions.
- Find the mean and standard deviation of the heights of the Statistics 101 Females.
 - Find the maximum and minimum height of the Statistics 101 Females.
 - Use R to make a normal quantile plot of the height values. Print this plot and include it with your assignment. According to the plot, do you think the height of Statistics 101 Females has a normal distribution? Explain your answer.
 - Regardless of your answer to the previous question, let's assume the heights of Stat 101 Females do have a normal distribution. What do you think are reasonable values for μ and σ for this distribution? Explain your answer.
 - The height data for this problem are for females only. Explain why I did not include the males in these data.
7. The weekly amount of down time Y (in hours) for an industrial machine has a gamma distribution with $\alpha = 3$ and $\beta = 2$.
- Find the probability the machine will be down for more than 3 hours.
 - For 95% of all weeks, the machine will be down less than how many hours?
 - The loss (in dollars) to the industrial operation as a result of down time is the function $L = 30Y + 2Y^2$. Find the expected loss.
8. In class, we learned that the parameters for the gamma distribution are the shape parameter α and the scale parameter β . Use R to generate 10,000 values from each of four gamma distributions. The four gamma distributions will all have $\beta = 2$, but will have $\alpha = 5, 20, 50, 100$, respectively. For each set of observed gamma values, calculate the mean and variance and obtain a histogram. Then write a paragraph explaining the effect of α on the shape, mean and variance of the gamma distribution.
9. Explosive devices used in mining operations produce nearly circular craters when detonated. The radii of these craters are exponentially distributed with a mean of 10 feet.
- Find the probability that one of these explosive devices will produce a radii between 6 and 12 feet.
 - Find the mean and variance of the areas produced by these explosive devices.
 - Find the probability that an explosive device will produce an area larger than 64π ft².
10. Let the random variable Y have an exponential distribution with mean β . Find $E(Y^r)$ for any positive integer r .