Statistics 341

Fall 2008 - Homework Assignment #5

Due Wednesday, October 29

1. Suppose a continuous random variable Y possesses the density function

$$f(y) = \begin{cases} cy & 0 \leq y \leq 2 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the value of c that makes $f(y)$ a probability density function

   (b) Find $F(y)$.

   (c) Find $P(1 \leq Y \leq 2)$ using $F(y)$.

   (d) Find $P(0.5 \leq Y \leq 1.5)$ using $f(y)$.

   (e) Find $E(Y)$.

   (f) Find $V(Y)$.

2. A supplier of kerosene has a 150-gallon tank that is filled at the beginning of each week. His weekly demand shows a relative frequency behavior that increases steadily up to 100 gallons and then levels off between 100 and 150 gallons. If Y denotes weekly demand in hundreds of gallons, the relative frequency of demand can be modeled by

$$f(y) = \begin{cases} y & 0 \leq y \leq 1 \\ 1 & 1 < y \leq 1.5 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find $F(y)$.

   (b) Find $P(0 \leq Y \leq 0.5)$.

   (c) Find $P(0.5 \leq Y \leq 1.2)$.

   (d) Find $E(Y)$.

   (e) Find $V(Y)$.

3. A gas station operates two pumps, each of which can pump up to 10,000 gallons of gas in a month. The total amount of gas pumped at the station in a month is a random variable Y (measured in 10,000 gallons) with a probability density function given by

$$f(y) = \begin{cases} y & 0 < y < 1 \\ 2 - y & 1 \leq y < 2 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the probability that the station will pump between 8000 and 12,000 gallons in a particular month.

   (b) Find the expected number of gallons the station will pump in a particular month.

   (c) Find the variance of the number of gallons the station will pump in a particular month.

4. As a measure of intelligence, mice are timed when going through a maze to reach a reward of food. The time (in seconds) required for any mouse is a random variable Y with a density function given by

$$f(y) = \begin{cases} \frac{b}{y^2} & y \geq b \\ 0 & \text{elsewhere} \end{cases}$$

where b is the minimum possible time needed to traverse the maze.

(a) Show the $f(y)$ has the properties of a density function.

(b) Find $P(Y > b + c)$ for a positive constant c.

(c) Find the expected number of seconds required for a particular mouse to reach the reward.

(d) Find the variance of the number of seconds required for a particular mouse to reach the reward.