Statistics 341

Fall 2007 - Assignment #2

Due Friday, September 19, 2008

This assignment is worth a total of 100 points.

1. (9 pts) Problem 2.74

   (a) (3 pts) A and D independent?
   For A and D to be independent, $P(A \cap D) = P(A)P(D)$. $P(A) = 0.61$ and $P(D) = 0.3$. $P(A)P(D) = 0.183$ which is not equal to $P(A \cap D) = 0.2$. A and D are dependent.

   (b) (3 pts) B and D independent?
   For B and D to be independent, $P(B \cap D) = P(B)P(D)$. $P(B) = 0.3$ and $P(D) = 0.3$. $P(B)P(D) = 0.09$ which is equal to $P(A \cap D) = 0.09$. B and D are independent.

   (c) (3 pts) C and D independent?
   For C and D to be independent, $P(C \cap D) = P(C)P(D)$. $P(C) = 0.09$ and $P(D) = 0.3$. $P(C)P(D) = 0.027$ which is not equal to $P(C \cap D) = 0.01$. C and D are dependent.

2. (14 pts) Problem 2.77

   (a) (1 pt) $P(A) = 0.4$
   (b) (1 pt) $P(B) = 0.37$
   (c) (1 pt) $P(A \cap B) = 0.10$
   (d) (2 pts) $P(A \cup B) = 0.27 + 0.1 + 0.30 = 0.67$
   (e) (1 pt) $P(\overline{A}) = 1 - P(A) = 0.6$
   (f) (2 pts) $P(\overline{A \cup B}) = 0.33$
   (g) (2 pts) $P(\overline{A \cap B}) = 0.27 + 0.33 + 0.30 = 0.9$
   (h) (2 pts) $P(A|B) = 0.1/0.37$
   (i) (2 pts) $P(B|A) = 0.1/0.4$

3. (4 pts) Problem 2.79

   A and B being mutually exclusive events means that $P(A \cap B) = 0$. If the events are independent, $P(A \cap B) = P(A)P(B)$. Since $P(A \cap B) = 0$, the only way for independence to hold if for $P(B) = 0$ ($P(A) > 0$), which means that B must be the empty set. Otherwise, mutually exclusive events cannot be independent events.

4. (6 pts; 2 pts for calculating the probability of seeing the ad, 4 pts for the rest) Problem 2.111

   We have two things going on here: seeing the ad and purchasing the product. Potential customers can see the ad either through a magazine or through TV or both. The probability they see the ad in a magazine is 0.02 and through TV is 0.20. Some customers will see the ad in both places. This probability is 0.01. So the probability a potential customer will see the ad is $0.02 + 0.20 - 0.01 = 0.21$.

   Now we can use the box idea to finish the problem. The probability a potential customer will see the ad is 0.21 and not see the ad is 0.79.

	See Ad	Not See Ad
Purchase		
Not Purchase		
	0.21	0.79

Out of the 0.21 that have seen the ad, 1/3 will purchase. This is $0.21(1/3) = 0.07$ total. Out of the 0.79 that have not seen the ad, 1/10 will purchase. This is $0.79(1/10) = 0.079$ total.

	See Ad	Not See Ad	
Purchase	0.07	0.079	0.149
Not Purchase	0.14	0.711	0.851
	0.21	0.79	

The probability a randomly selected potential customer will purchase is $0.07 + 0.079 = 0.149$.

5. (9 pts) Problem 2.114

   When the person is telling the truth, the lie detector has probabilities of a positive reading $= 0.1$ and a negative reading $= 0.9$. When the person is not telling the truth, the lie detector has probabilities of a positive reading $= 0.95$ and a negative reading $= 0.05$.

   (a) (2 pts) Since the events are independent, this is the multiplication of the two probabilities. $0.1(0.95) = 0.095$

   (b) (2 pts) Since the events are independent, this is the multiplication of the two probabilities. $0.9(0.95) = 0.855$

   (c) (2 pts) Since the events are independent, this is the multiplication of the two probabilities. $0.1(0.05) = 0.005$

   (d) (3 pts) This is the sum of the answers to part (a), (b) and (c). $0.095 + 0.855 + 0.005 = 0.955$. You could also calculate this probability using the complement event, a negative reading for both people. Since the events are independent, this is the multiplication of the two probabilities. $0.9(0.05) = 0.045$ You can then get the probability of the desired event by taking $1 - 0.045 = 0.955$.

6. (4 pts) Problem 2.129

   The partition of the sample space S is by gender. There are 15/20 females and 5/20 males. Of the females, 70% will react positively and out of the males, 40% will react positively. Here is the information in the box format.

	Female	Male	
React +	0.525	0.10	0.625
React -	0.225	0.15	0.375
	0.75	0.25	

   Given a negative reaction, the probability of a male is $0.15/0.375 = 0.4$

7. (4 pts) Problem 2.130

   The partition is lung cancer or not lung cancer. The probabilities of the two partition events are 0.0004 and 0.9996. Of the 0.0004 lung cancer, 22% worked at the shipyard or $(0.0004)(0.22) = 0.000088$ total. Of the 0.9996 not lung cancer, 0.14 worked at the shipyard or $(0.9996)(0.14) = 0.139944$ total. Here is the information in the box format.

	Lung Cancer	No Lung Cancer	
Work in Shipyard	0.000088	0.139944	0.140032
Don't Work in Shipyard	0.000312	0.859656	0.859968
	0.0004	0.9996	

Given they worked in the shipyard, the probability of lung cancer is 0.000088/0.140032 = 0.000628

8. (12 pts) In 2006, cyclist Floyd Landis won the sports' premier event, the Tour de France. After completing the race, the World Anit-Doping Agency released test results which indicated that Landis had high testosterone levels in his urine after one of the race stages. After further litigation, the International Court of Arbitration for Sport upheld the test results, stripped Landis of the Tour de France win, and banned him from professional cycling for two years.

   During the course of the 2006 Tour de France, Landis submitted 8 urine specimens for analysis. The test results on these 8 specimens depend on the accuracy of the testing procedures. For this problem, assume Landis was NOT doping and that each test result is independent of the others. Let the probability of a correct no-doping test result given the assumption that Landis was not doping to be the value p.

   (a) (9 pts; 3 pts for each) What is the probability all 8 urine tests would come back with the correct no-doping test result if the value of p = 0.95? What about when the value of p = 0.99? What about for a general value p?

   Since each test result is independent of all others, the probability of 8 correct no-doping test results if the value of $p = 0.95$ is $0.95^8 = 0.6634$.

   If the value of $p = 0.99$, the probability of 8 correct no-doping test results is $0.99^8 = 0.9227$.

   For a general value of p, the probability of 8 correct no-doping test results is p^8.

   (b) (3 pts) Determine the value of p so that the probability all 8 urine tests would come back with the correct no-doping test result is 0.99.

   We want to find the value of p so that $p^8 = 0.99$. Solving for p gives 0.9987.

9. (18 pts) Now assume that Landis WAS doping. Let the probability of a correct doping test result given the assumption that Landis was doping to be the value r. (Each test result is still independent of all others).

   (a) (6 pts; 2 pts for 0; 3 pts for 1; 1 pt for total) What is the probability that zero or one test out of the 8 would have a correct doping test result if the value of r = 0.9?

   Chance of getting 0 out of 8 correct doping test results would be $0.1^8 = 0.00000001$. Chance of getting 1 out of 8 correct doping test results would be $8(0.1)^7(0.9) = 0.00000072$. Getting either 0 or 1 would be the sum of these two probabilities = 0.00000073.

   (b) (6 pts; 2 pts for 0; 3 pts for 1; 1 pt for total) What is the probability that zero or one test out of the 8 would have a correct doping test result if the value of r = 0.95?

   Chance of getting 0 out of 8 correct doping test results would be 0.05^8. Chance of getting 1 out of 8 correct doping test results would be $8(0.05)^7(0.95)$. Getting either 0 or 1 would be the sum of these two probabilities $= 5.976563X10^{-9}$.

(c) (6 pts; 2 pts for 0; 3 pts for 1; 1 pt for total) Write a general formula using r for the probability that zero or one test out of the 8 would have a correct doping test result.
Chance of getting either 0 or 1 test of 8 with the correct doping test result would be

$$r^8 + 8(1 - r)(r^7)$$

10. (4 pts) In a class there are four freshman boys, six freshman girls, and six sophomore boys. How many sophomore girls must be present if gender and class are to be independent when a student is selected at random?

There are a couple of ways to approach this problem. One is using the box idea below.

	Freshman	Sophomore	
Boys	4	6	10
Girls	6	x	6 + x
	10	6 + x	16 + x

For example, the probability of a boy is $10/(16 + x)$ and the probability of a freshman is $10/(16 + x)$ and the probability of a freshman boy is $4/(16 + x)$. For these events to be independent, the probability of the intersection (freshman boy) must be equal to the multiplication of the probability of freshman and the probability of boy.

$$\left(\frac{10}{16 + x}\right)\left(\frac{10}{16 + x}\right) = \frac{4}{16 + x}$$

Solving the equation above for x gives $x = 9$.

11. (16 pts) This problem is based on the famous (at least in statistics and mathematics) Monty Hall problem. Monty Hall was the host of a game show called "Let's Make a Deal". In the game, he presents a contestant with three doors. Behind one door is something valuable, like a new car, and behind the other two doors are cheap prizes, like a skillet or a can of soup. Monty Hall begins the game by asking the contestant to pick a door. He then shows the contestant what is behind one of the two doors the contestant did not pick. Behind this door is one of the cheap prizes.

At this point, Monty Hall asks the contestant if they want to stay with the door they originally selected or switch to the other door.

(a) (2 pts) Which strategy is better, stay or switch, or does it matter? Explain your answer.
Answers will vary. At this point, you may not know or realize the strategy should be to switch doors. If you switch doors, the probability of winning will be 2/3. If you stay with your original door, the probability of winning will be 1/3.

(b) (2 pts) Go to the following web address

http://www.dcity.org/braingames/3doors/default.html

Scroll down the page to the game. Pick a strategy (either stay or switch) and play the game 100 times using the strategy you selected. What proportion of times did you win the valuable prize with this strategy?
Answers will vary. If you switch, you will probably win around 66 times out of 100. If you stay, you will probably win around 33 times out of 100.

(c) (2 pts) Play the game another 100 times using the other strategy (if you switched before, now you stay, or vice versa.) What proportion of times did you win the valuable prize with the other strategy?

Answers will vary. If you now stay, you will probably win around 33 times out of 100. If you now switch, you will probably win around 66 times out of 100.

(d) (2 pts) What do you think the probability of winning is if you stay with your original door? What do you think the probability of winning is if you switch doors? Explain your answer.

Again, you may not yet realize the probabilities are 2/3 for switching and 1/3 for staying. However, you should realize that the probability for switching is higher than the probability for staying due to your 100 trials of each above.

(e) (8 pts; 6 pts for filling out table; 2 pts for the probabilities) A table can be used to determine all the possible outcomes of the game. The theoretical probability of winning with each strategy can then be calculated. Copy the table below to your homework assignment and fill in the missing areas. Use the table to determine the probability of winning when you switch doors and the probability of winning if you stay with your original door. (Are you surprised?)

Prize is Behind	You Pick	Stay	Switch
A	A	Win	Lose
A	B	Lose	Win
A	C	Lose	Win
B	A	Lose	Win
B	B	Win	Lose
B	C	Lose	Win
C	A	Lose	Win
C	B	Lose	Win
C	C	Win	Lose

Of the 9 possibilities, when you switch you will win 6 out of 9 times. When you stay you will win 3 out of 9 times.