

Gamma Distribution - Section 4.6

The Gamma Distribution is used to model continuous data whose values are positive and have a probability histogram that is skewed to the right. The properties of the gamma distribution are

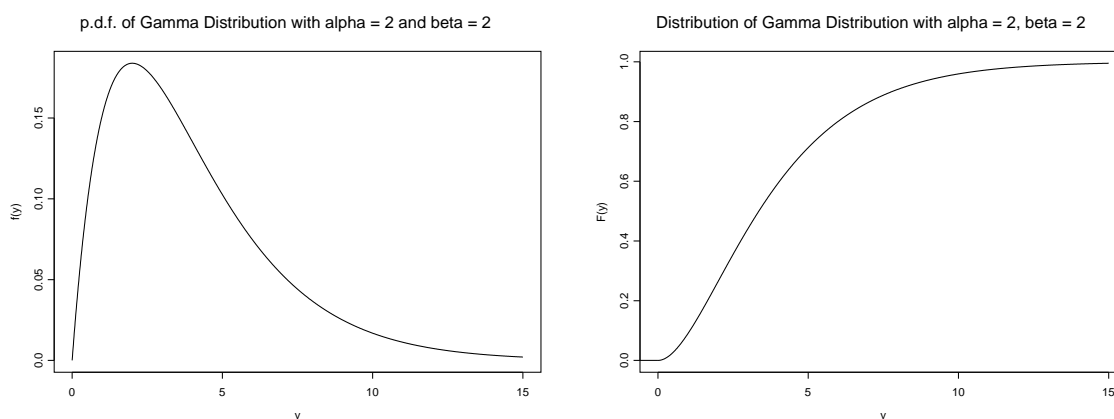
- The parameters for the gamma distribution are $\alpha > 0$ and $\beta > 0$. The α parameter is referred to as the shape parameter and the β parameter is referred to as the scale parameter for reasons will be investigating.
- The probability density function for the gamma distribution is

$$f(y) = \frac{y^{\alpha-1} e^{-y/\beta}}{\beta^\alpha \Gamma(\alpha)} \quad 0 \leq y < \infty$$

where $\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy$.

- In most circumstances, the distribution function $F(y)$ does not have a closed form solution. Therefore, the probabilities for the gamma distribution must be obtained from tables or from a statistical software package.

Here are graphs of the probability density function and the distribution function of a Gamma distribution with $\alpha = 2$ and $\beta = 2$.



- The theoretical mean of the gamma distribution is

$$\mu = E(Y) = \alpha\beta$$

- The variance of the gamma distribution is

$$\sigma^2 = V(Y) = \alpha\beta^2$$

Working with the gamma distribution in R.

To find the probability $P(Y \leq y)$ the command in R is

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pgamma(y, shape = alpha, scale = beta)
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To find the value of y such that $P(Y \leq y) = p$ the command in R is

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qgamma(p, shape = alpha, scale = beta)
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To generate observations from a gamma distribution the command in R is

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rgamma(numobs, shape = alpha, scale = beta)
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where numobs is the number of observed values you would like to generate.

Problems.

1. Four week summer rainfall total in a section of the Midwest have approximately a gamma distribution with $\alpha = 1.6$ and $\beta = 2.0$. The unit of measurement is inches.
 - (a) Find the probability that the rainfall total for this particular four weeks in the summer will be greater than 5 inches.
 - (b) Find the mean amount of rainfall for this particular four weeks in the summer.
 - (c) Find the variance of the rainfall amounts for this particular four weeks in the summer.
2. Annual incomes for heads of household in a section of a city have approximately a gamma distribution with $\alpha = 1000$ and $\beta = 20$. The unit of measurement is dollars.
 - (a) Find the mean and variance of the incomes for heads of households in this section of the city.
 - (b) The median income is defined as the income where 50% make more and 50% make less. Find the median income.
 - (c) The first quartile income is defined as the income where 25% make less and 75% make more. Find the first quartile income.
 - (d) The third quartile income is defined as the income where 75% make less and 25% make more. Find the third quartile income.
 - (e) Between what two incomes do the middle 50% of heads of households in this section of the city make?