Expectations for cont. r.v.

Note Title 10/27/2008

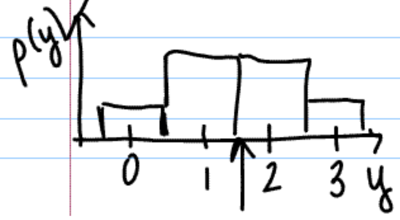
$E(Y) = \mu$ theoretical

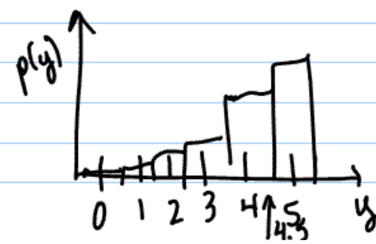
mean value for an infinite # of obs. of r.v.

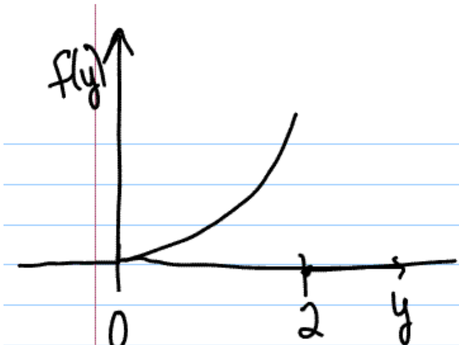
observed mean $\bar{y} \approx$ theoretical mean μ with many obs.

μ as a balancing point.

binomial with $p = 0.9$ $n = 5$







$$f(y) = \begin{cases} \frac{3}{8} y^2 & 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Discrete $\mu = \sum_y y \cdot p(y)$

Cont. $\mu = \int_{-\infty}^{\infty} y \cdot f(y)\, dy$

Discrete $E(g(Y)) = \sum_{y} g(y) \cdot p(y)$

Cont. $E(g(Y)) = \int_{-\infty}^{\infty} g(y)\, f(y)\, dy$

Ex. $f(y) = \begin{cases} \frac{3}{8} y^2 & 0 \le y \le 2 \\ 0 & \text{elsewhere} \end{cases}$

$E(Y)$, $E(Y^2)$

$E(Y) = \int_0^2 y \cdot \frac{3}{8} y^2 \, dy = \frac{3}{32} y^4 \Big|_0^2 = \frac{48}{32} - 0 = \boxed{\frac{48}{32}}$

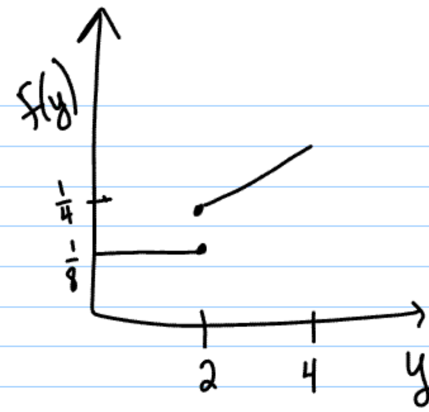
$E(Y^2) = \int_0^2 y^2 \, \frac{3}{8} y^2 \, dy = \frac{3}{40} y^5 \Big|_0^2 = \boxed{\frac{96}{40}}$

$V(Y) = E\left[(Y-\mu)^2\right] = E(Y^2) - (E(Y))^2$

$- \mu^2$

$= \frac{96}{40} - \left(\frac{48}{32}\right)^2$

$$f(y) = \begin{cases} 0 & y < 0 \\ \frac{1}{8} & 0 \le y \le 2 \\ \frac{y}{8} & 2 \le y \le 4 \\ 0 & y > 4 \end{cases}$$



$E(Y)$, $E(Y^2)$

$$\int_{-\infty}^{\infty} y \cdot f(y)\, dy = \int_{-\infty}^{0} \cancel{0\, dy} + \int_0^2 y \cdot \frac{1}{8}\, dy + \int_2^4 y \cdot \frac{y}{8}\, dy + \int_4^{\infty} \cancel{0\, dy}$$

$$= \boxed{\frac{62}{24}} \checkmark$$

$$E(Y^2) = \int_0^2 y^2 \frac{1}{8}\, dy + \int_2^4 y^2 \frac{y}{8}\, dy = \boxed{\frac{47}{6}}$$

$$V(Y) = E(Y^2) - \left(E(Y)\right)^2$$

$$\frac{47}{6} - \left(\frac{62}{24}\right)^2$$

Expectation Rules

C = Constant

1. $E(c) = c$
2. $E(c\,g(Y)) = c\,E(g(Y))$
3. $E(g_1(Y) + g_2(Y) + \cdots + g_k(Y)) = E(g_1(Y)) + E(g_2(Y)) + \cdots + E(g_k(Y))$
4. $V(Y) = E(Y^2) - (E(Y))^2$
5. $V(aY+b) = a^2\,V(Y)$