Statistics 341
Exam #1
September 26, 2008

Name ________________________________

INSTRUCTIONS: Read the questions carefully and completely. Answer each question and show all your work in the space provided. Credit cannot be given if work is not shown. Good luck!

1. (12 pts) Two events A and B are such that $P(A) = 0.25$, $P(B) = 0.35$ and $P(A \cup B) = 0.5$.

   Given this information, you should be able to put the events A and its complement and B and its complement into a box. This will enable you to correctly determine the probabilities of the events A and B and its complements. Here is the box.

	B	$\overline{B}$	
A	0.1	0.15	0.25
$\overline{A}$	0.25	0.5	0.75
	0.35	0.65	1

   Find the probability that

   (a) (3 pts) A and B both occur.
   $P(A \cap B) = 0.1$

   (b) (3 pts) either A occurs or B occurs, but not both.
   $P(A \cap \overline{B}) + P(\overline{A} \cap B) = 0.15 + 0.25 = 0.4$

   (c) (3 pts) neither A nor B occurs.
   $P(\overline{A} \cap \overline{B}) = 0.5$

   (d) (3 pts) B occurs given that A occurs.
   $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.25} = 0.4$

2. (5 pts) Crates of eggs are inspected for blood clots by randomly selecting 5 eggs from the crate without replacement and examining their contents. If all five eggs are found to contain no blood clots, the crate is shipped. Otherwise, the crate is rejected. If the crate contains a total of 100 eggs of which 8 have blood clots, what is the probability the crate will be shipped?

There are two ways to approach this problem. One is to think about a large box with 100 eggs, separated into two parts, the 8 with blood clots, and the 92 without blood clots. There are $\binom{100}{5}$ ways to select the 5 eggs for inspection. In order to ship the crate, the 5 eggs must be selected from the 92 without blood clots. There are $\binom{92}{5}$ ways to select the 5 eggs for inspection from the 92 without blood clots. The probability of the event that the crate will be shipped is

$$P(\text{crate shipped}) = \frac{\binom{92}{5}}{\binom{100}{5}} = 0.6532$$

The other way to approach this problem is through the use of dependent events. The drawing of the eggs is done without replacement. On each draw from the crate, an egg without blood clots must be drawn. Any draw of an egg with blood clots will cause the crate to be rejected. We need the probability that the first 5 draws will all be eggs without blood clots. Let A_i be the event that the ith draw is an egg without blood clots. This probability is

$$\begin{aligned} P(A_1 \cap A_2 \cap A_3 \cap A_4 \cap A_5) &= P(A_1)P(A_2|A_1)\cdots P(A_5|A_1 \cap A_2 \cap A_3 \cap A_4) \\ &= \left(\frac{92}{100}\right)\left(\frac{91}{99}\right)\left(\frac{90}{98}\right)\left(\frac{89}{97}\right)\left(\frac{88}{96}\right) = 0.6532 \end{aligned}$$

3. (10 pts) A deck of cards consists of 52 cards. There are 4 suits and 13 values. They are

Suit	Values
Hearts	A 2 3 4 5 6 7 8 9 10 J Q K
Diamonds	A 2 3 4 5 6 7 8 9 10 J Q K
Spades	A 2 3 4 5 6 7 8 9 10 J Q K
Clubs	A 2 3 4 5 6 7 8 9 10 J Q K

The Ace (A) can be the low card (below the 2) or the high card (above the King (K)).

(a) (5 pts) Draw 3 cards from the deck without replacement. What is the probability a heart, a heart and a diamond are selected from the deck in that order?

The order of the selection matters, so you must use dependent events to calculate this probability. Let A_1 be the event of a heart on the first draw, A_2 be the event of a heart on the second draw, and A_3 be the event of a diamond on the third draw.

$$\begin{aligned} P(A_1 \cap A_2 \cap A_3) &= P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2) \\ &= \left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\left(\frac{13}{50}\right) = 0.0153 \end{aligned}$$

(b) (5 pts) Draw 3 cards from the deck without replacement. What is the probability of obtaining two hearts and one diamond from the deck in any order?

The order of the selection doesn't matter, so there are a couple of ways you could approach this problem. One way is to think of the cards in their suits. You need to select three cards from the deck; there are $\binom{52}{3}$ ways to do this. Then you need to select 2 cards from the hearts and one card from the diamonds. There are $\binom{13}{2}\binom{13}{1}$ ways to do this. This makes the probability

$$\frac{\binom{13}{2}\binom{13}{1}}{52 choose 3} = 0.0459$$

Another way to think about this problem is to use your answer for part (a) above. The event in part (a) assumes an order for the three cards: heart, heart, diamond. You could have two other orders for these two suits: heart, diamond, heart or diamond, heart, heart. Each of the three possibilities has the same probability, and so the probability of the event is $3 * 0.0153 = 0.0459$.

4. (11 pts) In a certain community, the probability that a randomly selected adult over the age of 50 has diabetes is 0.15. Health services in this community will correctly diagnosis 85 percent of all persons with diabetes as having the disease. They will also incorrectly diagnose 5 percent of all persons without diabetes as having the disease.

   There are two sets of events for each person in this problem. One event is whether or not the person has diabetes. This event is the partition B and the complement of B. The second event is whether or not the person is diagnosed as having diabetes. This event is A and the complement of A. We only have information about A conditioned on the partitioning events B and the complement of B. To fill in the box, we have

	Has Diabetes	Does not have Diabetes	
Diagnosed with Diabetes	0.1275	0.0425	0.17
Not Diagnosed with Diabetes	0.0225	0.8075	0.83
	0.15	0.85	1

   Find the probability that

   (a) (5 pts) the health services will diagnose an adult over 50 as having diabetes.
   $P(\text{Diagnosed with Diabetes}) = 0.1275 + 0.0425 = 0.17$

   (b) (6 pts) a person over 50 diagnosed by the health services as having diabetes actually has the disease.
   $P(\text{Has Diabetes}|\text{Diagnosed with Diabetes}) = \frac{0.1275}{0.17} = 0.75$

5. (12 pts) A missile protection system consists of n radar sets operating independently. Each set has a probability of 0.9 of detecting a missile entering a zone that is covered by all of the units.

   For each part below, let A_i be the event that the ith radar unit detects the missile.

   (a) (4 pts) If $n = 3$, what is the probability that all units will detect the missile?

   In this case, we will need to have the event A_i occur for each of the three radar units.

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3) = 0.9^3 = 0.729$$

   (b) (4 pts) If $n = 5$, what is the probability that at least one unit will detect the missile?

   At least one means that one out of the 5 must detect the missile, or 2 out of the 5 must detect the missile, or 3 out of 5, etc. It is much easier to find the complement of the event, which is that none of the units will detect the missile. In this case, we need to have the event $\overline{A}_i$ occur for each of the five radar units.

$$P(\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 \cap \overline{A}_4 \cap \overline{A}_5) = P(\overline{A}_1)P(\overline{A}_2)P(\overline{A}_3)P(\overline{A}_4)P(\overline{A}_5) = 0.1^5 = 0.00001$$

   This makes the probability of the desired event $1 - 0.1^5 = 0.99999$.

   (c) (4 pts) How many radar sets n would be necessary if the probability of at least one of them detecting the missile must be 0.999?

   From part (b) above, the probability of at least one of the radar sets detecting the missile is $1 - 0.1^n$. We must find n so that this probability is 0.999.

   $1 - 0.1^n = 0.999$ means that $0.1^n = 0.001$ which means that $n = 3$.

6. (10 pts) For this problem, refer to the R code below. You do not need to type this code into R.

```
days<- c(1:365)
bdbreaks<- c(0:365) + 0.5

maxcounts<- rep(0,10000)

for (i in 1:10000){
bdays26<- sample(days, 26, replace = T)
bdcounts<- hist(bdays26, breaks = bdbreaks, plot = F)$counts
maxcounts[i]<- max(bdcounts)
}

table(maxcounts)
maxcounts
   1    2    3    4
4062 5739  196    3
```

(a) (5 pts) Describe the random event being simulated with the R code above.

The random event is the basis of the birthday problem. We are simulating birthdays for 26 people in the room. We are then keeping track of how many birthdays occur on each day of the year. If the value of maxcounts is 1, this means that no one shared a birthday; if the value of maxcounts is 2, this means that at least one birthday was shared by 2 people; if maxcounts is 3, this means that at least one birthday was shared by 3 people; etc.

(b) (5 pts) Define a probability associated with this random event and give an estimate of the probability using the R code above.

The birthday problem is the probability that at least two people share a birthday. We can get an estimate of this probability using the 10,000 trials in the R code. The estimate is $(5739 + 196 + 3)/10000 = 0.5938$.