Relationship Between Events

Note Title 9/10/2008

Events $A, \bar{A}$ Events $B, \bar{B}$

	B	$\bar{B}$
A	$A \cap B$	$A \cap \bar{B}$
$\bar{A}$	$\bar{A} \cap B$	$\bar{A} \cap \bar{B}$

4 intersections are mutually exclusive.

$P(A \cup B)$, $P(A)$, $P(\bar{A})$

$P(A) + P(\bar{A})$ $P(B) + P(\bar{B})$

$$P(A \cup \bar{A}) = P(A) + P(\bar{A})$$

$$P(S) = P(A) + P(\bar{A})$$

$$1 = P(A) + P(\bar{A})$$

$$1 - P(\bar{A}) = P(A)$$

Any event A $P(A) = 1 - P(\bar{A})$

$$A = \underbrace{(A \cap B) \cup (A \cap \bar{B})}$$

$$P(A) = P(A \cap B) + P(A \cap \bar{B}) \leftarrow$$

$$P(B) = P(A \cap B) + P(\bar{A} \cap B) \leftarrow$$

$$P(A \cup B) = P(A \cap B) + P(A \cap \bar{B}) + P(\bar{A} \cap B)$$

$$= P(A \cap B) + P(A) - P(A \cap B) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) - P(A \cap B)$$

Conditional Probability

Exp. E_i in S. $P(A)$ looking at $P(E_i)$ in A that are in S.

Exp. Event B occurred Given we know event B occurred, what's the prob. of A?

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

E_i in B. A looking at E_i in A that are also in B.

Dependent & Independent Events

Independent Events – Knowledge of the occurrence of B doesn't change probability of A.

$* P(A|B) = P(A) \longrightarrow$ If and only if

$* P(B|A) = P(B) \longrightarrow$ conditions

$P(A\cap B)/P(B) = P(A)$

$* \boxed{P(A\cap B) = P(A)P(B)} \longrightarrow$

Dependent Events.

$P(A|B) \neq P(A)$

$P(B|A) \neq P(B)$

$P(A\cap B) \neq P(A)P(B)$

Independent Events — $P(A \cap B) = P(A)P(B)$

Dependent Events — $P(A \cap B) = P(B) \cdot P(A|B)$

$= P(A) \cdot P(B|A)$

1.

35 W	24 R	39 B

100

$\Downarrow$ 3

$A = \{1^{st} \text{ draw is } B\}$ - $P(A) = \frac{40}{100}$

$B = \{2^{nd} \text{ draw is } R\}$ - $P(B|A) = \frac{25}{99}$

$C = \{3^{rd} \text{ draw is } B\}$ - $P(C|A \cap B) = 39/98$

$$P(A \cap B \cap C) = \frac{40}{100} \cdot \frac{25}{99} \cdot \frac{39}{98} =$$

$A = \{1^{st} \text{ draw is } W\}$ - $P(A) = \frac{35}{100}$

$B = \{2^{nd} \text{ draw is } W\}$ - $P(B|A) = 34/99$

$C = \{3^{rd} \text{ draw is } W\}$ $P(C|A \cap B) = 33/98$

$P(A\cap B\cap C) = \frac{35}{100} * \frac{34}{99} * \frac{33}{98}$

Counting Methods

100 → 3 → $\binom{100}{3}$

35 W | 65 Other → 3 → $\binom{35}{3}$

$A = \{3W \text{ out of } 3\}$

$P(A) = \binom{35}{3} \Big/ \binom{100}{3}$

(2.) $A = \{1^{st} \text{ set Detects aircraft}\}$ $P(A) = 1 - 0.02 = 0.98$

$B = \{2^{nd} \text{ set Detects } \text{''} \}$ $P(B) = 0.98$

$C = \{3^{rd} \text{ set } \text{''} \ \text{''} \}$ $P(C) = 0.98$

(a) $P(\bar{A}\cap\bar{B}\cap\bar{C}) = P(\bar{A})P(\bar{B})P(\bar{C}) = (0.02)^3 = 0.000008$

(b) $P(A\cap B\cap C) = P(A)P(B)P(C)$

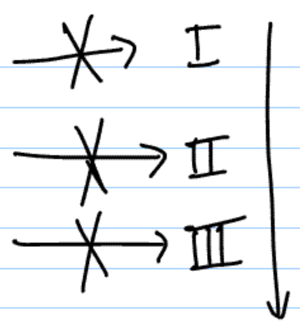
$= (0.98)^3 =$

Independent Events

$P(A\cap B)=0.25$	0.25	$P(A)=0.5$
0.25	0.25	$P(\bar{A})=0.5$
$P(B)=0.5$	$P(\bar{B})=0.5$	

$$P(A\cap B)=P(A)P(B)$$
$$P(A\cap \bar{B})=P(A)P(\bar{B})$$
$$P(\bar{A}\cap B)=P(\bar{A})P(B)$$
$$P(\bar{A}\cap \bar{B})=P(\bar{A})P(\bar{B})$$

(3.)



system failure

$P(\text{failure of either I, II or III}) = 0.01$

Failures are independent.

$$P(\text{no system failure}) = 1 - P(\text{system failure})$$
$$1 - P(\text{all 3 fail}) = 1 - P(\text{I fails})\cdot P(\text{II fails})\cdot P(\text{III fails})$$
$$= 1-(0.01)^3 = \boxed{0.999999}$$

4. a. $1/n$

b. $P(\text{C.P. on } 2^{nd} \text{ try}) = P(\text{I.P. on } 1^{st} \text{ try} \cap \text{C.P. on } 2^{nd} \text{ try})$

$= P(\text{I.P. on } 1^{st} \text{ try}) \cdot P(\text{C.P. on } 2^{nd} \text{ try} \mid \text{I.P. on } 1^{st} \text{ try})$

$= \left(\frac{\cancel{n-1}}{n}\right)\left(\frac{1}{\cancel{n-1}}\right) = \frac{1}{n}$

$P(\text{C.P. on } 3^{rd} \text{ try}) = P(\text{I.P. on } 1^{st} \cap \text{I.P. on } 2^{nd} \cap \text{C.P. on } 3^{rd} \text{ try})$

$P(\text{I.P. on } 1^{st})\, P\left(\begin{matrix}\text{I.P. on}\\ 2^{nd}\end{matrix} \,\middle|\, \begin{matrix}\text{I.P.}\\ \text{on}\\ 1^{st}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{C.P.}\\ 3^{rd}\end{matrix} \,\middle|\, \text{I.P. on } 1^{st} + 2^{nd}\right)$

$\left(\frac{\cancel{n-1}}{n}\right)\left(\frac{\cancel{n-2}}{\cancel{n-1}}\right)\left(\frac{1}{\cancel{n-2}}\right) = \frac{1}{n}$

$P(\text{C.P. on } i^{th} \text{ try}) = \frac{1}{n}$

5. $\longrightarrow \text{I} \longrightarrow \text{II}$

$P(\text{D, I}) = 0.1 \qquad P(\text{D, II} \mid \text{D, I}) = 0.5$

$P(\text{D,I} \cap \text{D,II}) = P(\text{D, I}) \cdot P(\text{D, II} \mid \text{D, I}) = 0.1(0.5) = 0.05$