Probability for Equally Likely Events (Counting)

Note Title 9/8/2008

E_i are simple events that make-up S.

Event $A = (E_1 \cup E_2 \cup \cdots \cup E_n)$

So $P(A) = \sum_{i=1}^{n} P(E_i)$

If all events are equally likely, $P(E_i) = \frac{1}{N}$,

$N = \#$ of E_i in S.

$$P(A) = \sum_{i=1}^{n} P(E_i) = \sum_{i=1}^{n} \frac{1}{N} = \frac{n}{N} = \frac{\#\text{ of } E_i \text{ in } A}{\#\text{ of } E_i \text{ in } S}$$

Counting Methods for n & N.

<u>m×n Rule</u>

If m possible outcomes for Event 1
If n possible outcomes for Event 2
Then $m \times n$ possible outcomes for combination of Event 1 & Event 2

Extends to multiple events. Think Tree Diagram.

Permutations

How many ways can you order r objects selected from n objects?

n objects = 1 2 3 4

select r=3 of them.

123	124	134	234
132	142	143	243
213	214	314	324
231	241	341	342
312	412	413	423
321	421	431	432

24 ways total. $\underset{1^{st}}{\underline{4}} * \underset{2^{nd}}{\underline{3}} * \underset{3^{rd}}{\underline{2}} = 24$

$n = 50 \quad r = 3 \qquad \underline{50} * \underline{49} * \underline{48} = \frac{50!}{47!} = \frac{n!}{(n-r)!}$

Combinations

How many ways can you select r objects from n objects?

P = select & order

C = select

In example above = 24 permutations; 4 combinations

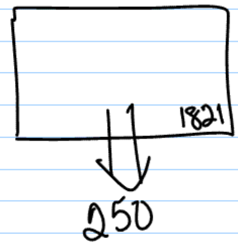
$$\# \text{ of ways} = \frac{n!}{(n-r)!\,r!} = \binom{n}{r} = {}_nC_r$$

this is notation I will use.

number of orderings of each selected set.

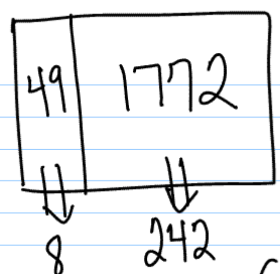
Probability Examples

1. Experiment – Selecting 250 songs from 1821 songs



$$\# \text{ of ways} = \binom{1821}{250}$$

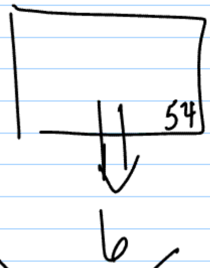
$A = \{$ 8 out of 250 songs will be Shania Twain songs $\}$



$$\# \text{ of ways} = \binom{49}{8}\binom{1772}{242}$$

$$P(A) = \frac{\binom{49}{8}\binom{1772}{242}}{\binom{1821}{250}}$$

(2.) Experiment - Selecting 6 balls from 54.

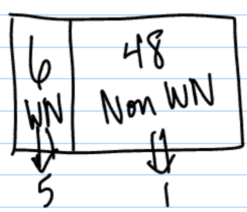


$\#$ of ways $= \binom{54}{6}$

(a) $A = \{$ match all 6 numbers $\}$

$\#$ of ways $= 1$ $\quad$ $P(A) = \dfrac{1}{\binom{54}{6}}$

(b) $A = \{$ match exactly 5 numbers $\}$



$\#$ of ways $= \binom{6}{5}\binom{48}{1}$

$$P(A) = \frac{\binom{6}{5}\binom{48}{1}}{\binom{54}{6}}$$

(c) $A = \{\text{match exactly 4 numbers}\}$

6 WN	48 Non WN
4	2

$\#\text{ of ways} = \binom{6}{4}\binom{48}{2}$

$$P(A) = \frac{\binom{6}{4}\binom{48}{2}}{\binom{54}{6}}$$

52
5

$\#\text{ of } E_i \text{ in } S = \binom{52}{5} = 2{,}598{,}960$

(a) $A = \{\text{all 5 cards from same suit}\}$

$\#\text{ of } E_i \text{ in } A = 4 * \binom{13}{5} = 5{,}148$

$$P(A) = \frac{5148}{2598960} = 0{,}00198$$

(b.) $A = \{4 \text{ of a Kind}\}$

$\#$ of E_i in A = $13 * 48$

$$= \underset{\substack{\text{4 of} \\ \text{a Kind}}}{\overset{\text{4 of a Kind}}{\binom{13}{1}}} * \underset{\substack{\text{value} \\ \text{for extra} \\ \text{card}}}{\overset{\text{extra card}}{\binom{12}{1}}} * \underset{\substack{\text{extra} \\ \text{card}}}{\binom{4}{1}}$$

$$= 624$$

$P(A) = 624/2598960 = 0.0002$

(c.) $A = \{3 \text{ of a Kind} + \text{one Pair}\}$

$\#$ of E_i in A = $13 * 12 * \binom{4}{3} * \binom{4}{2}$

$$= 3744$$

$P(A) = \dfrac{3744}{2598960} = 0.00144$

(d) $A = \{2 \text{ Pairs}\}$

2 of one value + 2 of another value + 1 card of another value.

$$\#\text{ of } E_i \text{ in } A = \underbrace{\binom{13}{2}}\ \underbrace{\binom{4}{2}\binom{4}{2}\underbrace{44}_{\binom{11}{1}\cdot\binom{4}{1}}}$$

$$P(A) = \frac{123552}{2598960} = 0.0475$$

(e.) $A = \{\text{one Pair}\}$

Pair of 1 value + 1 card of another value + 1 card of another value + 1 card of another value

$$\underbrace{\binom{13}{3} * \binom{10}{1}}_{\text{values of singles} \quad \text{value of pair}} \quad \underbrace{\binom{4}{2} * \binom{4}{1}\binom{4}{1}\binom{4}{1}}$$

$$\underbrace{\binom{13}{1}_{\text{Value of pair}} * \binom{12}{3}_{\text{Values of singles}}} = 1{,}098{,}240$$

$$P(A) = \frac{1098240}{2598960} = 0.42257$$