Textbook Problems HMWK #3

**3.1** When the health department tested private wells in a county for two impurities commonly found in drinking water, it found that 20% of the wells had neither impurity, 40% had impurity $A$, and 50% had impurity $B$. (Obviously, some had both impurities.) If a well is randomly chosen from those in the county, find the probability distribution for $Y$, the number of impurities found in the well.

**3.5** A problem in a test given to small children asks them to match each of three pictures of animals to the word identifying that animal. If a child assigns the three words at random to the three pictures, find the probability distribution for $Y$, the number of correct matches.

**3.30** Suppose that $Y$ is a discrete random variable with mean $\mu$ and variance $\sigma^2$ and let $X = Y + 1$.

**a** Do you expect the mean of $X$ to be larger than, smaller than, or equal to $\mu = E(Y)$? Why?

**b** Use Theorems 3.3 and 3.5 to express $E(X) = E(Y + 1)$ in terms of $\mu = E(Y)$. Does this result agree with your answer to part (a)?

**c** Recalling that the variance is a measure of spread or dispersion, do you expect the variance of $X$ to be larger than, smaller than, or equal to $\sigma^2 = V(Y)$? Why?

**d** Use Definition 3.5 and the result in part (b) to show that

$$V(X) = E\{[(X - E(X)]^2\} = E[(Y - \mu)^2] = \sigma^2;$$

that is, $X = Y + 1$ and $Y$ have *equal* variances.

**3.31** Suppose that $Y$ is a discrete random variable with mean $\mu$ and variance $\sigma^2$ and let $W = 2Y$.

**a** Do you expect the mean of $W$ to be larger than, smaller than, or equal to $\mu = E(Y)$? Why?

**b** Use Theorem 3.4 to express $E(W) = E(2Y)$ in terms of $\mu = E(Y)$. Does this result agree with your answer to part (a)?

**c** Recalling that the variance is a measure of spread or dispersion, do you expect the variance of $W$ to be larger than, smaller than, or equal to $\sigma^2 = V(Y)$? Why?

**d** Use Definition 3.5 and the result in part (b) to show that

$$V(W) = E\{[W - E(W)]^2\} = E[4(Y - \mu)^2] = 4\sigma^2;$$

that is, $W = 2Y$ has variance four times that of $Y$.

**3.32** Suppose that $Y$ is a discrete random variable with mean $\mu$ and variance $\sigma^2$ and let $U = Y/10$.

**a** Do you expect the mean of $U$ to be larger than, smaller than, or equal to $\mu = E(Y)$? Why?

**b** Use Theorem 3.4 to express $E(U) = E(Y/10)$ in terms of $\mu = E(Y)$. Does this result agree with your answer to part (a)?

**c** Recalling that the variance is a measure of spread or dispersion, do you expect the variance of $U$ to be larger than, smaller than, or equal to $\sigma^2 = V(Y)$? Why?

**d** Use Definition 3.5 and the result in part (b) to show that

$$V(U) = E\{[U - E(U)]^2\} = E[.01(Y - \mu)^2] = .01\sigma^2;$$

that is, $U = Y/10$ has variance .01 times that of $Y$.

**3.33** Let $Y$ be a discrete random variable with mean $\mu$ and variance $\sigma^2$. If $a$ and $b$ are constants, use Theorems 3.3 through 3.6 to prove that

**a** $E(aY + b) = aE(Y) + b = a\mu + b$.

**b** $V(aY + b) = a^2V(Y) = a^2\sigma^2$.

**3.34** The manager of a stockroom in a factory has constructed the following probability distribution for the daily demand (number of times used) for a particular tool.

| $y$ | 0 | 1 | 2 |
|---|---|---|---|
| $p(y)$ | .1 | .5 | .4 |

It costs the factory \$10 each time the tool is used. Find the mean and variance of the daily cost for use of the tool.