Binomial Distribution

Note Title 10/3/2008

Sample from Population with replacement.

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trials are identical. Probability (Success) = $\boxed{p}$

Two outcomes for each trial Success (Category of interest) Failure (not the category of interest)

$\boxed{Y = \#\text{ of successes in } \boxed{n} \text{ trials}}$

n & p are called parameters of dist.

S S F F F

$$P(Y=5) = P(S \cap S \cap S \cap S \cap S) = P(S)P(S)P(S)P(S)P(S) = p^5$$

$$P(Y=0) = P(F \cap F \cap F \cap F \cap F) = P(F)P(F)P(F)P(F)P(F) = (1-p)^5$$

$$P(Y=1) = 5 \cdot P(F \wedge F \wedge F \wedge F \wedge S) = 5 P(F) P(F) P(F) P(F) P(S)$$
$$= 5 \cdot p (1-p)^4$$

$$P(Y=4) = 5 \cdot P(S \wedge S \wedge S \wedge S \wedge F) = 5 \cdot P(S)^4 \cdot P(F)$$
$$= 5 p^4 (1-p)$$

$$P(Y=2) = 10 \cdot P(S \wedge S \wedge F \wedge F \wedge F) = 10 \cdot P(S)^2 P(F)^3$$
$$= 10 \cdot p^2 (1-p)^3$$

$$P(Y=3) = 10 \cdot P(S \wedge S \wedge S \wedge F \wedge F) = 10 \cdot P(S)^3 P(F)^2$$
$$= 10 \cdot p^3 (1-p)^2$$

y	$p(y)$
0	$(1-p)^5$
1	$5 \cdot p (1-p)^4$
2	$10 \cdot p^2 (1-p)^3$
3	$10 \cdot p^3 (1-p)^2$
4	$5 \cdot p^4 (1-p)$
5	p^5

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

$$\left(p + (1-p)\right)^n = \sum_{y=0}^{n} \binom{n}{y} p^y (1-p)^{n-y}$$

$$p(y) = \binom{n}{y} p^y (1-p)^{n-y} \qquad y = 0, 1, 2, \ldots, n$$

$$E(Y) = \mu = np$$

$$V(Y) = \sigma^2 = np(1-p)$$

$$E(Y) = \sum_y y \cdot p(y)$$

$$= \sum_{y=0}^{n} y \cdot \binom{n}{y} p^y (1-p)^{n-y}$$

$$= \sum_{y=1}^{n} \left[y \cdot \binom{n}{y} \right] p^y (1-p)^{n-y}$$

$$y \cdot \frac{n!}{y! (n-y)!} \qquad (y-1)!$$

$$= \sum_{y=1}^{n} \frac{n!}{(y-1)!(n-y)!} p^y (1-p)^{n-y}$$

$$n(n-1)!$$

$$= \sum_{y=1}^{n} \frac{n(n-1)!}{(y-1)!\,(n-y)!} p^y (1-p)^{n-y}$$

$$= \sum_{y=1}^{n} n \binom{n-1}{y-1} p^y (1-p)^{n-y}$$

$$= np \sum_{y=1}^{n} \binom{n-1}{y-1} p^{y-1} (1-p)^{n-y}$$

$z = y-1$

$$= np \left[\sum_{z=0}^{n-1} \binom{n-1}{z} p^z (1-p)^{[n-(z+1)]} \right] \quad n-1+z$$

$$= np \cdot 1 = np$$

$V(Y) = np(1-p)$ for a given n

$V(Y)$ is maximized when $p = 0.5$

6. $p = 0.85$; $n = 17$ $\quad Y = \#$ of made free throws

(a) $P(Y=17) = \binom{17}{17}(0.85)^{17}(0.15)^{0} = 0.0631$

(b) $P(0 \le Y \le 10) = \sum_{y=0}^{10} \binom{17}{y}(0.85)^{y}(0.15)^{17-y}$

$= 0.0083$

(c) $np = 17(0.85) = 14.45$

(d) $\sigma = \sqrt{np(1-p)} = \sqrt{17(0.85)(0.15)}$

$= 1.47$

(e) The probability gets small at 10 free throws made or less.

7. $n = 5$, $p = 0.95$

(a) $P(Y=5) = (0.95)^{5} = 0.7738$

(b) $E(Y) = np = 5(0.95) = 4.75$

(c) $P(Y \leq 2) = 0.0012$

(d) P(system detects aircraft) is the same for both systems.

$P(\text{system detects}) = 1 - (0.05)^5 = 0.9999997$

with 5 radar sets with $p = 0.95$.

$P(\text{system detects}) = \boxed{0.9999997 = p}$ with just one radar set.