

Binomial Distributions

A discrete random variable is said to have a binomial distribution if

- There are a fixed number of independent and identical trials, n . This means that each trial can be thought of as a draw from a population, where the drawing is done with replacement.
- Each of the n trials has two possible outcomes, success and failure.
- The probability of success on each of the n trials is the same, p . Therefore, the probability of failure on each of the n trials is $1 - p = q$.
- The random variable Y is defined as the total number of successes in the n trials.
- The parameters for a binomial random variable Y are the number of trials n and the probability of success on each trial p .
- The probability distribution function of the binomial random variable Y is

$$p(y) = \binom{n}{y} p^y (1-p)^{n-y} \quad \text{for } y = 0, 1, \dots, n$$

- The theoretical mean of the binomial random variable Y is

$$\mu = E(Y) = np$$

- The theoretical variance of the binomial random variable Y is

$$\sigma^2 = V(Y) = np(1-p) = npq$$

Working with binomial random variables in R.

- Finding probabilities.

To find a probability $P(Y = y) = p(y)$ for a single value y , the command in R is

```
dbinom(y,n,p)
```

To find the probability $P(Y \leq y)$, use the sum command to add up all $p(y)$ values for y between and including 0 and y .

```
sum(dbinom(0:y,n,p))
```

To find the probability $P(y_1 \leq Y \leq y_2)$, use the sum command to add up all $p(y)$ values for y between and including y_1 and y_2 .

```
sum(dbinom(y1:y2,n,p))
```

To find the probability $P(Y \geq y)$, use the sum command to add up all $p(y)$ values for y between and including y and n

```
sum(dbinom(y:n,n,p))
```

Finally, to get a list of all $p(y)$ values for a particular binomial random variable Y , use the `dbinom` command to list all $p(y)$ for values of y between and including 0 and n .

```
dbinom(0:n,n,p)
```

- Generating observed values.

You can also use R to generate observed values from a binomial distribution with n and p . Generating a large number of observed values is helpful in studying the characteristics of the distribution of the random variable Y . The command is

```
rbinom(numobs, n, p)
```

where `numobs` is the number of observed values you would like to generate.

Problems.

1. How is the probability distribution function $p(y)$ derived?
2. Show that $p(y)$ has the two properties of a probability distribution function.
3. Derive the formula for the expected number of successes in n trials.
4. Use R to generate 10,000 observations of a binomial random variable with $n = 20$ and $p = 0.25$. Make a histogram of your observations and determine the mean, variance, and five number summary of your data. Use this information to describe your distribution.
5. Now use R to generate 10,000 observations of a binomial random variable with $n = 20$ and $p = 0.75$. Make a histogram of your observations and determine the mean, variance and five number summary of your data. How does this distribution compare to the distribution from problem 5?
6. From previous experience, it is known that the number of free throws a basketball player makes in a game behaves like a binomial random variable. For a particular basketball player, his probability of making a single free throw is 0.85. In a particular game, this basketball player attempts 17 free throws.
 - (a) Find the probability this player will make all 17 free throws attempted.
 - (b) Find the probability this player will make 10 or less free throws of the 17 attempted.
 - (c) Find the mean number of free throws made in this game.
 - (d) Find the standard deviation of the number of free throws made.
 - (e) At what point would you conclude that this player is having an “off” night?
7. A collection of 5 radar sets, functioning independently are used to detect the presence of aircraft in a given area. The probability that a radar set will detect a given aircraft is 0.95.
 - (a) Find the probability all 5 radar sets will detect an aircraft.
 - (b) Find the average number of radar sets that will detect an aircraft.
 - (c) Find the probability that two or fewer radar sets will detect the aircraft.
 - (d) Suppose instead of 5 radar sets, we have 1 radar set. What does the probability that the radar set will detect a given aircraft have to be for the two systems to be equivalent?