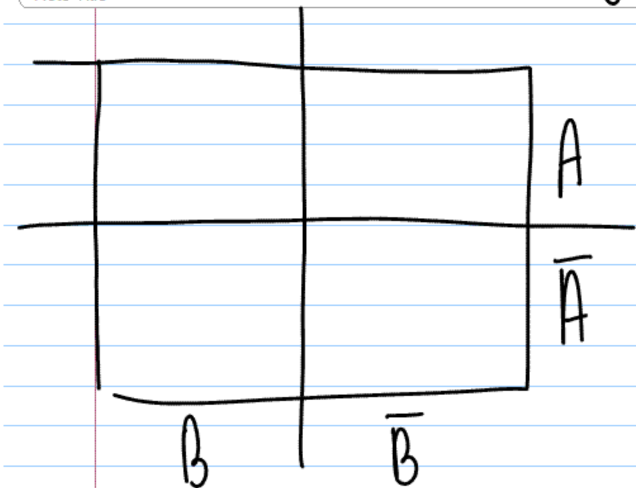
Conditional Probabilities & Bayes Theorem

Note Title 9/15/2008



$B, \bar{B}$ are called a partition of S.

B_1, B_2, B_3, B_4 are also a partition.

$P(A|B) + P(A|\bar{B})$

Medical testing

$B = \{\text{Have HIV}\}$

$\bar{B} = \{\text{Do not have HIV}\}$

$A = \{\text{test positive for HIV}\}$

	$B\ (0.01)$	$\bar{B}\ (0.99)$
A	$0.99(0.01)$ 0.0099	$0.99(0.001)$ 0.00099
$\bar{A}$	0.0001	0.98901

$P(B) = 0.01$

$P(A|B) = 0.99 \quad P(A|\bar{B}) = 0.001$

$$P(A\cap B) = P(B)\cdot P(A|B) = 0.01(0.99) = 0.0099$$

$$P(A\cap \bar{B}) = P(\bar{B})\cdot P(A|\bar{B}) = 0.99(0.001) = 0.00099$$

$$P(A) = P(B)P(A|B) + P(\bar{B})P(A|\bar{B})$$

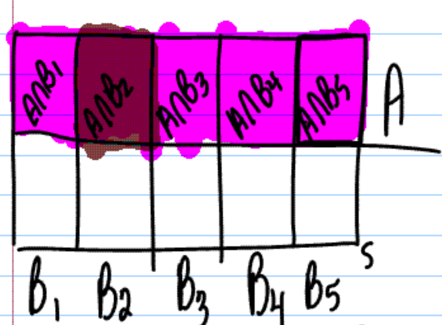
$$= 0.0099 + 0.00099 = 0.01089$$

$$P(B|A) = \boxed{\frac{P(A\cap B)}{P(A)}} = \frac{0.0099}{0.01089} = 0.\overline{90}$$

<u>Bayes Theorem</u>: Let $B_1, B_2, \ldots, B_k$ be a partition of the sample space S. Then

$$P(B_i|A) = \frac{P(B_i)\cdot P(A|B_i)}{\sum_{j=1}^{k} P(B_j)P(A|B_j)}$$

$$= \frac{P(A\cap B_i)}{P(A)}$$



$$P(A\cap B_5) = P(B_5)P(A|B_5)$$

1. Disease I & II

$A = \{\text{Have Disease I}\}$ $\quad B = \{\text{Have Disease II}\}$

$\bar{A}$ $\quad \bar{B}$

	B	$\bar{B}$	
A	0.03	0.07	$A(0.1)$
$\bar{A}$	0.12	0.78	$\bar{A}(0.9)$
	$B(0.15)$	$\bar{B}(0.85)$	

(a) $P(A \cup B) = 0.12 + 0.03 + 0.07 = 0.22$
$= 0.1 + 0.15 - 0.03 = 0.22$

(b) $P(A \cap B \mid A \cup B) = \dfrac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)} = \dfrac{P(A \cap B)}{0.22} = \dfrac{0.03}{0.22} = 3/22$

2.

0.032	0.06	$D(0.092)$	$P(D \mid I) = 0.08$
0.368	0.54	$ND(0.908)$	$P(D \mid II) = 0.1$
$I(0.4)$	$II(0.6)$		

$P(I \cap D) = P(I) \cdot P(D \mid I)$ $\quad P(II \cap D) = P(II) \cdot P(D \mid II)$

$$P(ND) = 0.908 = 1 - P(D)$$
$$= 1 - \left[P(I)\cdot P(D|I) + P(II)\,P(D|II)\right]$$
$$= 1 - 0.092 = 0.908$$

3.

	Strike 0.6	No Strike 0.4	
On time 0.55	0.6(0.35) 0.21	0.4(0.85) 0.34	On time 0.55
Not on time 0.45	0.39	0.06	Not on time 0.45

$$P(\text{Strike}|\text{Ontime}) = \frac{0.21}{0.55} = \boxed{0.38}$$

4.

	MI 0.35	MII 0.25	MIII 0.4
Defective 0.0215	0.35(0.02) 0.007	0.25(0.01) 0.0025	0.4(0.03) 0.012
Non-Defective 0.9785	0.343	0.2475	0.388

$$\frac{0.012}{0.0215} = \boxed{0.558}$$

6.

0.25 (0.99) 0.2475	(0.75)(0.17) 0.1275	Fails Test 0.3750	
0.0025	0.6225	Doesn't Fail Test 0.6250	
Emits	Doesn't Emit		
0.25	0.75		

0.3750

$$\frac{0.2475}{0.3750} = 0.66$$