Chapter 6

The Standard Deviation as a Ruler and the Normal Model

At the end of the chapter, you should be able to

- Compare observations from two different quantitative variables.
- Determine which quantitative variables could be modeled using the normal distribution.
- Apply the 68-95-99.7 Rule to any quantitative variable with a normal distribution.

At the end of this chapter, you should be able to

- Find percentile or area values for any given observation from a normal distribution.
- Find the value of an observation when given a percentile or area value from the normal distribution.

Comparing Two Variables

- ACT Score
- SAT Score
- Different scale
- Measuring similar values
- How do you compare them?

Example 1

Example 2

Standardizing Observations

- y = observation of quantitative variable
- How does the value of y relate to the mean value?
- How does the value of y for this quantitative variable relate to another observation of a different quantitative variable?

Standardizing Variables

$$z = \frac{y - \bar{y}}{s}$$

- z has no units (just a number)
- Puts observations on same scale.
  - Mean (center) at 0.
  - Standard deviation (spread) of 1.
- Does not change overall shape of the distribution.

Standardizing Variables

- z = # of standard deviations observation is away from mean.
  - Negative z – observation is below mean.
    - Ex. z = -2
    - Ex. z = -0.5
  - Positive z – observation is above mean.
    - Ex. z = 2
    - Ex. z = 0.5

Example 1

Example 1

Example 2

Example 2

Distributions and Standardizing

- Standardizing
  - Allows you to make comparisons of observations between different variables.
  - Without the distribution information, you still don't know anything about the percentile value of your observation.
  - This percentile value depends on the distribution.

Models for Distributions

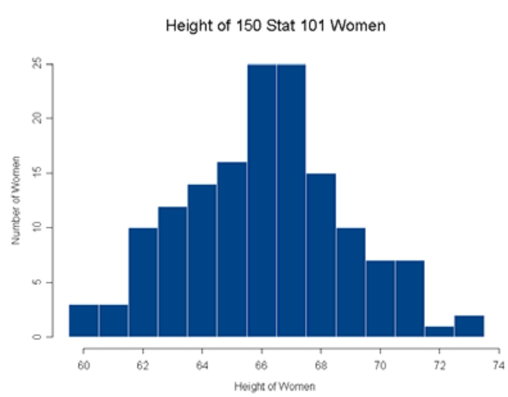
- Different Models According to Different Aspects of Distributions.
  - Shape
  - Center
  - Spread

Normal Distribution

- Shape
  - Unimodal
  - Symmetric
  - Bell-Shaped
- Mean (μ)
- Standard Deviation (σ)

Connection to Data

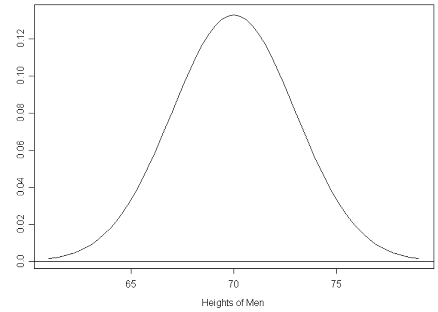
- No Data Distribution has an exact normal distribution.
- Many distributions are close enough.
- How do you know?
  - Histogram
  - Normal Quantile Plot (Lab)



A Closer Look at Normal Distributions

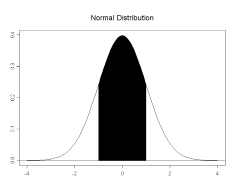
- Two parameters (not calculated)
  - Mean μ (pronounced "meeoo")
    - Locates center of curve
    - Splits curve in half
  - Standard deviation σ (pronounced "sigma")
    - Controls spread of curve
    - Ruler of distribution
- Write as N(μ,σ)

Example – Height of Men



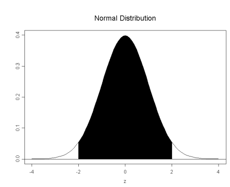
68-95-99.7 Rule for Normal Distributions.

- Approx. 68% of observations are within 1 σ of the mean μ.



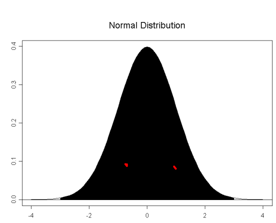
68-95-99.7 Rule for Normal Distributions.

- Approx. 95% of observations are within 2 σ of the mean μ.



68-95-99.7 Rule

- 99.7% of observations are within 3 σ of the mean μ.



Standard Normal Distribution

- Puts all normal distributions on same scale.

$$z = \frac{y - \mu}{\sigma}$$

- z has center (mean) at 0
- z has spread (standard deviation) of 1

Standard Normal Distribution

- z = # of standard deviations away from mean μ
  - Negative z = number is below the mean
  - Positive z = number is above the mean
- Written as N(0,1)

Standardizing

- Y is N(70,3). Standardize y = 68.

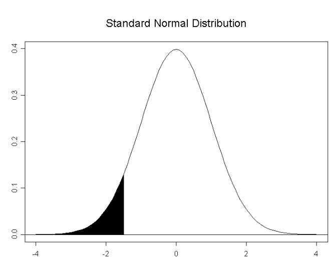
Standardizing

- Y is N(70,3). Standardize y = 74.

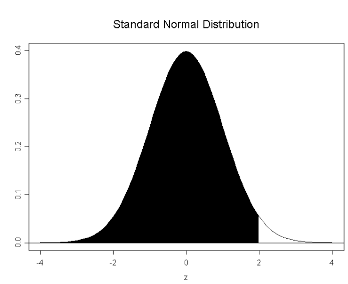
Normal Values Table

- Going between standard values and percentiles on the normal distribution
- Table gives proportion of curve below a particular standardized value z (the percentile for the value z)
  - z values range from −3.90 to 3.90
  - Row – ones and tenths place for z.
  - Column – hundredths place for z.

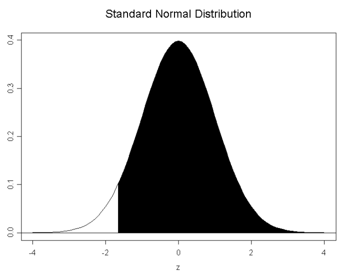
Normal Percentile for z = -1.50



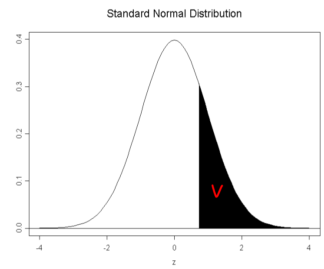
Normal Percentile for z = 1.98



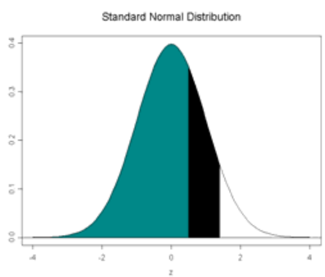
Proportion greater than z = -1.65



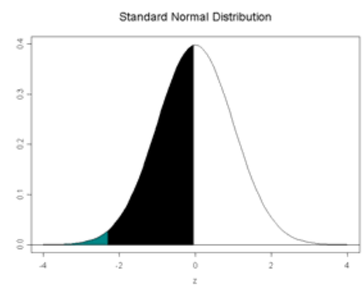
Proportion greater than z = 0.73



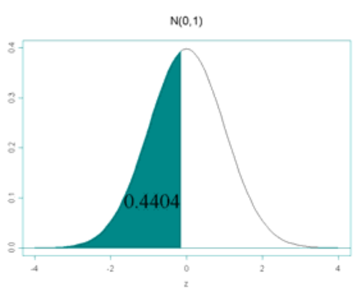
Proportion between z = 0.5 and z = 1.4



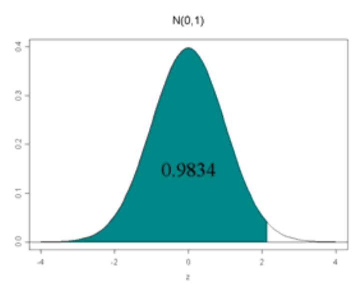
Proportion between z = -2.3 and z = -0.05



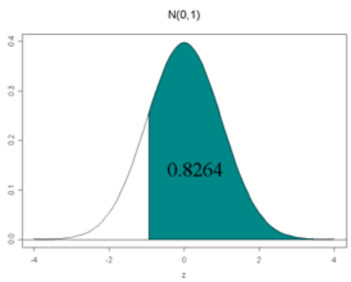
Finding z from a given area



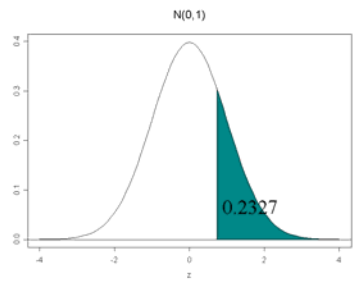
Finding z from a given area



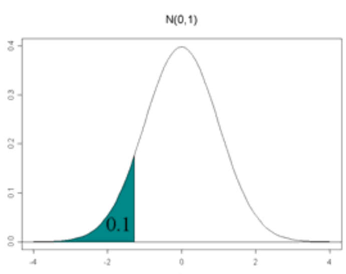
Finding z from a given area



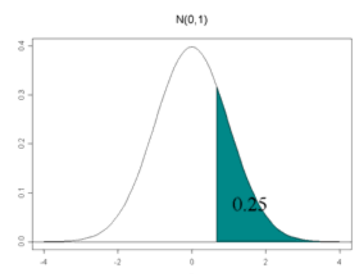
Finding z from a given area



Finding z from a given area



Finding z from a given area

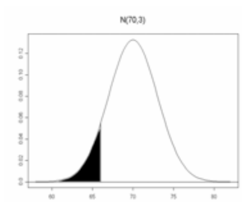


Example #1

- The height of men is known to be normally distributed with mean 70 and standard deviation 3.
  - Y is N(70,3)

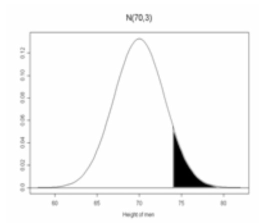
Example #1A

- What proportion of men are shorter than 66 inches?



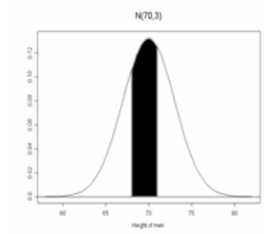
Example #1B

- What proportion of men are taller than 74 inches?



Example #1C

- What proportion of men are between 68 and 71 inches tall?



Example #1D

- What are the values of the median, Q1 and Q3 for the height of men?

Example #1D – Q1

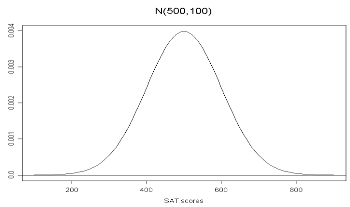
Example #1D – Q3

Example #2

- Scores on SAT verbal are known to be normally distributed with mean 500 and standard deviation 100.
- X is N(500,100)

Example #2A

- Your score was 650 on the SAT verbal test. What percentage of people scored better?



Example #2B

- What would you have to score to be in the top 5% of people taking the SAT verbal?