

A Study of Methods for Selecting the AT
Subtest in the DIMTEST Procedure

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DIMTEST procedure provides a hypothesis test of unidimensionality.

Split test into two subgroups of items.

- (dimensionality) Assessment Subtest (AT)
- (examinee) Partitioning Subtest (PT)

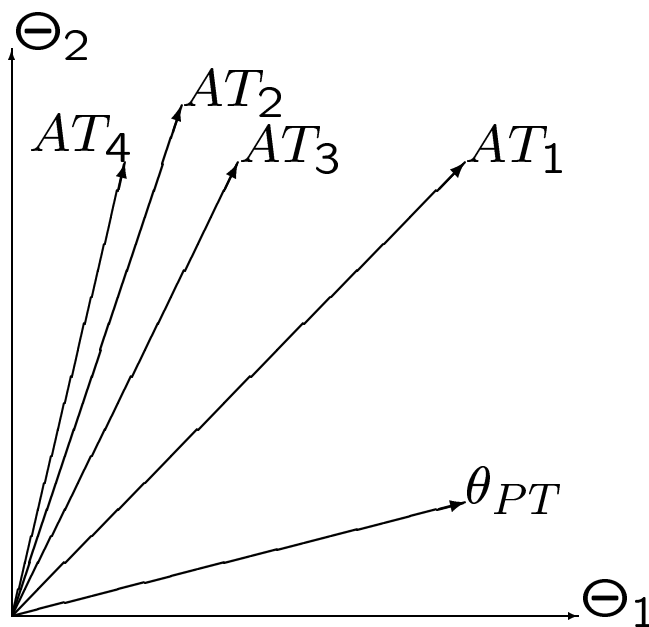
H_0 : The AT Subtest measures the same direction as the PT Subtest.

H_a : The AT Subtest measures a different direction than the PT Subtest. (The test is multidimensional.)

Selecting the AT Subtest.

Choose items for AT that are

1. Dimensionally Homogenous (Close together)
2. Dimensionally Distinct from PT (Far away from PT)



Current Ways to Choose AT

- Linear Factor Analysis Using Tetrachoric Correlations
- Non-linear Factor Analysis
- HCA/CCPROX Cluster Analysis
- DETECT
- Expert Opinion
- Etc.

Automated AT Selection

Would be nice to select AT automatically using some exploratory data method.

The only method that is currently programmed for this application is Linear Factor Analysis. (called FAC in DIMTEST software)

Using FAC to choose AT

- First suggested by Stout (1987)
- Refined by Nandakumar & Stout (1993)
- Froelich & Stout (2002) found reduced power for the DIMTEST procedure in some cases and no power in other cases using FAC
- They also found these reductions in DIMTEST power were largely eliminated when an “ideal” AT was used.

Numerous studies have shown linear factor analysis is not in general a good method for exploring dimensionality.

Another method is needed for AT Selection.

Reasonable Starting Place - Conditional Covariances

Idea of Conditional Covariances

Work of Zhang & Stout (1999)

Define the direction of best measurement of the PT Subtest, θ_{PT} .

For a multidimensional test, they found for items U_i and U_k

$$Cov(U_i, U_k | \theta_{PT})$$

- = 0, if U_i or U_k in same “direction” as θ_{PT}
- > 0, if U_i or U_k on same “side” of θ_{PT}
- < 0, if U_i or U_k on different “sides” of θ_{PT}

HCA/CCPROX (Roussos, Stout, & Marden, 1998)

- Hierarchical cluster analysis
- Uses conditional covariance item pair matrix to calculate proximity value
- Clusters are joined based on this proximity value
 - Small proximity - joined early in cluster analysis
 - Large proximity - joined later in cluster analysis

DETECT (Zhang & Stout, 1999)

- Searches for the best partitioning of items into clusters.
- Partitions are evaluated using the DETECT statistic (based on conditional covariance item pair matrix).
- Final DETECT partitioning of items has largest DETECT statistic.

Use HCA/CCPROX or DETECT to select AT?

Downsides to using HCA/CCPROX to select AT

- No indication which cluster to use as AT.
- Locked into hierarchical structure.

Downsides to using DETECT to select AT

- No indication which cluster to use as AT.
- When data are unidimensional, DETECT might find only one cluster.
- Clusters might not be a desirable size.

4 AT Selection Methods

Method I

- Select all item clusters from HCA/CCPROX with between 3 items and $1/3$ of the test items.
- Cluster from HCA/CCPROX = possible AT.
Rest of the items = Corresponding PT.
- Calculate DETECT statistic for each of these two cluster partitions.
- Partition with largest DETECT value = AT and PT subtests.

Method II

- Select all item clusters from HCA/CCPROX with between 3 items and $1/3$ of the test items.
- Cluster from HCA/CCPROX = possible AT. Rest of the items = Corresponding PT.
- Calculate DIMTEST statistic (not bias corrected) for each of these two cluster partitions.
- Partition with largest DIMTEST value = AT and PT subtests.

Method III

- Find best two cluster solution using DETECT.
- Smaller of the two clusters is AT; larger is PT.
- No limitation on the size of AT.

Method IV

- Select all item clusters from HCA/CCPROX with between 3 items and $1/3$ of the test items.
- Cluster from HCA/CCPROX = possible AT.

Method IV (cont.)

Select best PT to go with corresponding AT.

1. For each item (U_j) not in AT
 - (a) Calculate conditional covariance value (from DETECT) between U_j and each possible AT item (U_i).

- (b) Calculate the following statistic

$$\frac{-\sum_{i \in AT} cov(U_i, U_j)}{\sum_{i \in AT} |cov(U_i, U_j)|}$$

- (c) Statistic is between -1 and 1 for all U_j
 - (d) When statistic is negative, potential PT item is close to AT items.
 - (e) When statistic is positive, potential PT item is far from AT items.

Method IV (cont.)

2. Ignore as many items as possible with values less than a certain limit.
3. Remaining items are corresponding PT Subtest.

Which (AT, PT) partition is the best?

- Select the partition with the largest value of

$$\sum_{i < k \in AT} cov(U_i, U_k) - \sum_{j \in PT} \sum_{i \in AT} cov(U_i, U_j)$$

- Should give most homogenous AT and
- Should give PT that is farthest away from corresponding AT.

Type I Error Monte-Carlo Simulation Study

- 3PL model

$$P_i(\theta) = c_i + \frac{1 - c_i}{1 + \exp[-1.7a_i(\theta - b_i)]}$$

- $\theta \sim N(0, 1)$
- Item Parameters (a, b, c) estimated from three tests
 - 25 item ASVAB (Mislevy & Bock, 1984)
 - 40 item ACT math (Drasgow, 1987)
 - 80 item SAT verbal (Lord, 1968)

- Four examinee levels (750, 1000, 1500, 2000)
- Choose AT Subtest with (250, 350, 500, 750) examinees.
- Perform the DIMTEST hypothesis test with remaining examinees.
- Nominal Rate: $\alpha = 0.05$
- Number of complete trials = 100
- 4 cutoff values for Method IV = (0, 0.25, 0.5, 0.75)

Type I error results - Number of Rejections per 100

Linear Factor Analysis

Test	ASVAB	ACTM	SATV
J = 750	2	4	3
J = 1000	1	6	4
J = 1500	2	3	3
J = 2000	5	8	8

Method I

Test	ASVAB	ACTM	SATV
J = 750	5	9	3
J = 1000	2	1	5
J = 1500	3	1	2
J = 2000	4	7	5

Method II

Test	ASVAB	ACTM	SATV
J = 750	8	9	3
J = 1000	5	7	2
J = 1500	6	2	1
J = 2000	8	8	5

Method III

Test	ASVAB	ACTM	SATV
J = 750	3	2	4
J = 1000	3	2	4
J = 1500	3	5	3
J = 2000	1	2	2

Method IV - Cutoff Value 0

Test	ASVAB	ACTM	SATV
J = 750	5	2	2
J = 1000	5	3	5
J = 1500	6	2	2
J = 2000	10	12	10

Method IV - Cutoff Value 0.25

Test	ASVAB	ACTM	SATV
J = 750	6	2	2
J = 1000	4	4	4
J = 1500	5	2	1
J = 2000	7	15	11

Method IV - Cutoff Value 0.5

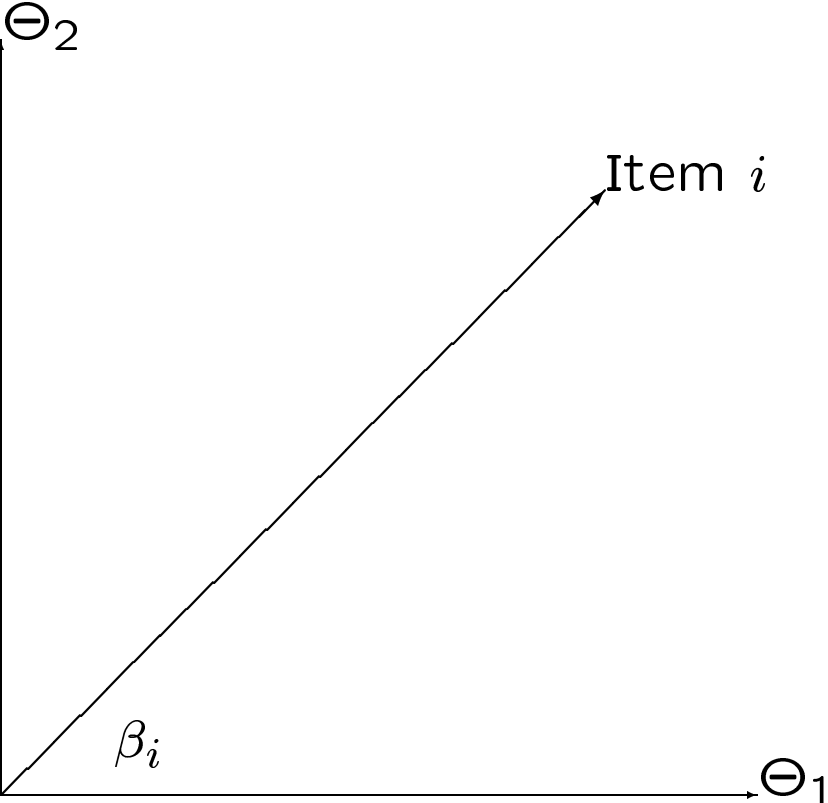
Test	ASVAB	ACTM	SATV
J = 750	6	3	3
J = 1000	5	3	4
J = 1500	5	3	1
J = 2000	9	15	11

Method IV - Cutoff Value 0.75

Test	ASVAB	ACTM	SATV
J = 750	6	2	2
J = 1000	3	3	3
J = 1500	4	2	1
J = 2000	8	15	10

Monte-Carlo Simulation Study - Power

β_i = angle between Θ_1 axis and direction of best measurement of item i .



Three different two-dimensional models.

Model I. Simple Structure

- 2/3 of items with $\beta_i = 0$.
- 1/3 of items with $\beta_i = 90$.

Model II. Approximate Simple Structure

- 2/3 of items with $0 < \beta_i < 30$
- 1/3 of items with $60 < \beta_i < 90$

Model III. No Structure

- $0^\circ \leq \beta_i \leq 90^\circ$

Two-Dimensional 3PL model

$$P_i(\boldsymbol{\theta}) = c_i + \frac{1 - c_i}{1 + \exp[-1.7\mathbf{a}_i^T \boldsymbol{\theta} + d_i]}$$

- $\boldsymbol{\theta} \sim N(0, 0, 1, 1, \rho)$ where $\rho = 0.3$ or 0.7 .
- Take (a_i, b_i, c_i) from ASVAB, ACTM, SATV

$$a_{i,1} = a_i \cos(\beta_i) \text{ and } a_{i,2} = a_i \sin(\beta_i)$$

$$d_i = -a_i * b_i$$

Same guessing parameters c_i

- Four levels of examinee size (750, 1000, 1500, 2000)
- AT Subtest chosen with (250, 350, 500, 750) examinees.
- Perform DIMTEST hypothesis test with remaining examinees.
- Nominal Rate: $\alpha = 0.05$.
- 4 cutoff values for Method IV = (0, 0.25, 0.5, 0.75)
- 100 simulations per trial.

Results - Correlation 0.3

Simple Structure Model

- all at or very near 100% rejection

Approximate Simple Structure

- all at or very near 100% rejection

No Structure

- All above 90% rejection.
- Most at or near 100% rejection.

Results - Correlation 0.7

Simple Structure

- All above 90% rejection.
- Most at or near 100%.

Approximate Simple Structure

Average rejection rate across examinee levels.

Test	ASVAB	ACTM	SATV
FAC	72.00	52.50	40.50
Method I	78.00	60.50	84.75
Method II	85.50	44.25	79.25
Method III	61.25	54.75	77.25
Method IV (0)	86.00	64.00	89.50
Method IV (0.25)	86.25	64.25	89.75
Method IV (0.5)	86.00	62.00	89.75
Method IV (0.75)	86.25	61.75	88.50

No Structure Model

ASVAB

Examinees	750	1000	1500	2000
FAC	17	22	46	49
Method 1	22	36	64	73
Method 2	24	42	65	81
Method 3	28	38	54	46
Method 4 (0)	24	45	75	88
Method 4 (0.25)	26	48	75	89
Method 4 (0.5)	23	47	77	90
Method 4 (0.75)	25	48	76	88

No Structure Model

ACTM

Examinees	750	1000	1500	2000
FAC	14	21	31	28
Method 1	10	27	62	86
Method 2	9	18	53	78
Method 3	21	25	38	45
Method 4 (0)	13	34	72	91
Method 4 (0.25)	12	37	70	94
Method 4 (0.5)	12	34	70	94
Method 4 (0.75)	12	34	72	93

No Structure Model

SATV

Examinees	750	1000	1500	2000
FAC	3	7	5	8
Method 1	15	28	55	83
Method 2	14	26	41	73
Method 3	14	28	27	36
Method 4 (0)	19	36	66	90
Method 4 (0.25)	24	35	62	89
Method 4 (0.5)	23	32	64	90
Method 4 (0.75)	23	33	63	92

Future Research

- Higher Dimensional Data
- Non-compensatory Data
- Further Development of Method IV