Stat 544 – Spring 2005
Homework assignment 6
Answer key

Problem 1

Suppose that we have have only two models: Model 1 and Model 2. Further, suppose that we iterate the Gibbs sampler ten times. The following table shows the value of the criterion C for each model and the value of I(A) at each iteration where I(A) denotes the indicator function of the set A=$\{C_1 - C_2 \leq 0\}$.

Iteration	C_1	C_2	I(A)
1	1	1	1
2	2	3	1
3	1	2	1
4	3	4	1
5	1	2	1
6	4	5	1
7	5	8	1
8	3	4	1
9	4	4	1
10	20	4	0
Mean	4.4	3.7	0.9

So, Model 1 is better than Model 2, since 90% of the time $C_1 - C_2 \leq 0$. However, if we use only the posterior means to compare both models we will conclude, erroneously, that Model 2 is better than Model 1.

Problem 2

a- WinBUGS codes
If you took the time to read the GeoBUGS manual, you may have found out that for WinBUGS to produce reasonable results it is recommended that spatial models be hierarchically centred. Hierarchically models that are not centred tend to produce incorrect results.

To hierarchically centre a model is to build some kind of scheme to make the observations Y_i conditionally independent. We have used the hierarchical representation of the model described in page 16 of the spatial data classnotes. Thus, instead of fitting the model

$$Y \sim N(\mu, \Sigma + v^2 I) \quad \text{with} \quad \mu = X\beta$$

we will fit the following model:

$$Y|\theta, W \sim N(X\beta + W, v^2 I) \quad (1)$$
$$W|\Sigma \sim N(0, \Sigma) \quad (2)$$

and since $\Sigma = w^2 H(\phi)$, with $H(\phi)_{ij} = \exp(-\phi d_{ij})$ we have that (2) can be rewritten as

$$W|\phi, w^2 \sim N(0, w^2 H(\phi))$$

Note that equation (1) guaranties that, under this setting, our observations are conditionally independent.

a.1- Declaring $H(\phi)$ matrix explicitly.

```
model{for(i in 1:N){ Y[i] ~ dnorm(mu[i], error.prec)
                     mu[i] <- beta + W[i]
                     muW[i] <- 0.0  }
    for(i in 1:N){ for(j in 1:N){
                H[i,j]<-sigmasq*exp(-phi*distances[i,j]) }}
error.prec ~ dgamma(0.1,0.1)
tausq <- 1/error.prec
beta ~ dnorm(0.0, 0.001)

W[1:N] ~ dmnorm(muW[], Omega[,])
spat.prec ~ dgamma(0.1,0.1)
sigmasq <- 1/spat.prec
phi ~ dunif(0,10)
Omega[1:N,1:N] <- inverse(H[,])
} # end model
```

a.2- Using the intrinsic `spatial.exp` function.

```
model{for (i in 1:N){ Y[i] ~ dnorm(mu[i], error.prec)
                      mu[i] <- beta + W[i]
                      muW[i] <- 0.0 }

W[1:N] ~ spatial.exp(muW[],latitud[],longitud[],spat.prec,phi,1)

error.prec ~ dgamma(0.1,0.1)
tausq <- 1/error.prec
beta ~ dnorm(0.0, 0.001)
spat.prec ~ dgamma(0.1,0.1)
sigmasq <- 1/spat.prec
phi ~ dunif(0,10)
} # end model
```

b - The following table shows results obtained with each one of the two different approaches. Results for the "direct" method are based on 6000 iterations of a two-chain run with a thinning factor of 3 (i.e. a sample size of 2400) after a burn-in period of 5000 iterations. Results for the "direct" method are based on 5000 iterations of a two-chain run with a thinning factor of 4 (i.e. a sample size of 2500) after a burn-in period of 5000 iterations. Thinning was applied to correct for autocorrelation.

				Percentiles		
Method	Parameter	Mean	std	2.5%	Median	97.5%
"direct"	β	5.000	0.543	3.965	4.992	6.071
	ϕ	4.994	2.893	0.268	4.977	9.764
	w^2	1.375	1.604	0.064	0.951	5.186
	v^2	1.205	1.195	0.066	0.871	4.262
"intrinsic"	β	4.983	0.530	3.987	4.981	6.112
	ϕ	5.088	2.841	0.445	5.091	9.751
	w^2	1.276	1.368	0.069	0.908	4.755
	v^2	1.297	1.334	0.072	0.952	4.580

c - Runtime in seconds based on a two-chain run

# iterations	Method	
per chain	direct	intrinsic
1000	1	25
2000	1	57
10000	9	282
250000	216	6619

Note that runtime is machine dependent, so you could have observed better or worse times.

Runtime was measured in WinBUGS, results are rounded to the nearest second.