

Stat 544 – Spring 2005

Homework assignment 4

Due on Wednesday, March 9 in TA's office by 5:00 pm

Problem 1

We wish to obtain draws y from a truncated (below) normal distribution with mean μ and variance σ^2 . Suppose that $\mu = 0$ and $\sigma^2 = 1$, so that

$$p(y|\mu, \sigma^2) \propto \exp\left(-\frac{y^2}{2}\right), \quad y \geq a$$

The naive method to sample from this distribution, is simply to draw values from the $N(0,1)$ and discard all draws that are less than a . This method may be effective if a is small relative to μ , but can get costly otherwise.

- a- In the example above, what is the average number of simulations from $N(0,1)$ required for each acceptance?
- b- A potential instrumental or proposal distribution in this case is the translated exponential distribution, with pdf

$$g_\alpha(z) = \alpha \exp[-\alpha(z - a)] \quad z \geq a$$

for some value of α .

- b.1- Derive the upper bound on the ratio $p/g_\alpha(z)$. Hint: There are two upper bounds on the ratio, depending on whether $\alpha > a$ or not.
- b.2- Find the value of α that minimizes the upper bound in each case.
- b.3- Given your results in part (b.2), propose an algorithm to sample values from a truncated normal distribution using the rejection sampling method. List all of the steps that are needed to carry out your algorithm.

Problem 2

Consider a simple linear regression model $y_i = \alpha + \beta x_i + e_i$, where $e_i \sim N(0, \sigma^2)$, and the x_i are known and fixed. Assume that the errors are independent.

Let the prior distributions for α and β be: $N(\alpha_0, \sigma_\alpha^2)$ and $N(\beta_0, \sigma_\beta^2)$, respectively, with parameters fixed and known. For σ^2 , let the prior distribution be an inverted scaled χ^2 distribution with known parameters.

We are interested in estimating the posterior distributions of α , β , and σ^2 .

- a- Write down the full probability model for this problem: likelihood, priors, and joint posterior.
- b- Write down the full conditional distributions for the parameters in the model, to implement a Gibbs sampler for this problem.
- c- In words, explain how you would go about implementing the Gibbs sampler
- d- Write down model portion of the WinBUGS code for this problem. Do not attempt to actually run the program, since you do not have any data.