Stat 544 – Spring 2005
Addendum to homework assignment 2

Due on Friday, February 11 in TA’s office by 5:00 pm

Problem 7

On part 3 of exercise 2 of homework 1 you were asked to write a WinBUGS code to generate 1000 draws from the posterior distribution of $\lambda \mid y$. Recall that

a- $y \mid \lambda \sim \text{Poisson}(\lambda)$

b- a Gamma(1,0.001) distribution was chosen as a prior for $\lambda$.

c- you observed the following values of $y$: 6, 6, 2, 6, 5, 8, 3, 6, 4, 5

The answer key states that the solution for this part is:

```r
model{ for (i in 1:N){ y[i] ~ dpois(lambda) } 
  lambda ~ dgamma(alpha,beta) 
}
list(y = c(6, 6, 2, 6, 5, 8, 3, 6, 4, 5),N=10,alpha=1,beta=0.001)
```

However, another possible solution may be this one:

```r
model{ for (i in 1:N){ y[i] ~ dpois(lambda) } 
  lambda ~ dgamma(alpha,beta) 
}
list(y = c(6, 6, 2, 6, 5, 8, 3, 6, 4, 5),N=10,alpha=1,beta=1000)
```

Please note that the difference between the answer key code and the alternative code is that in the former beta = .001 and in the latter beta = 1000. We claim that the answer key code is more accurate for the problem at hand and we ask for your help in arguing in favor of it.

To help you to defend our solution we suggest that you:

1- Answer the following question under each code:

What is your prior belief about $\lambda$?
2 - Obtain a sample of 1000 draws from the posterior predictive distribution under each code. In each case, compute and report the posterior mean, standard deviation, and percentiles 0.025, 0.50, and 0.975.

You may want to program this part using R.

After you have answered both questions please write a short explanation telling us why the answer key code is preferable over the second code.

Another thing that may help you is to remember that the parameterization that WinBUGS uses for a Gamma(\(\alpha, \beta\)) distribution is the following:

\[
X \sim \text{Gamma}(\alpha, \beta) \iff f(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x} \quad x \geq 0, \alpha, \beta > 0
\]

Solution

1 - Prior belief

Note that for the WinBUGS parameterization of the Gamma distribution we have

\[
E(\lambda) = \frac{\alpha}{\beta} \quad \text{Var}(\lambda) = \frac{\alpha}{\beta^2}
\]

Answer key code:

\[
E(\lambda) = 1000 \quad \text{Var}(\lambda) = 1,000,000
\]

Our prior belief about \(\lambda\) is that we do not have any prior information about \(\lambda\), i.e. we do not favor any specific interval of possible values of \(\lambda\) over any other interval. Note that by using a Gamma(1,0.001) we are assigning a non-informative prior to \(\lambda\).

Alternative code

\[
E(\lambda) = \frac{1}{1000} = 0.001 \quad \text{Var}(\lambda) = 0.000001
\]

Our prior belief is that \(\lambda\) is most certainly equal to 0.

2 - The following code can be use inside WinBUGS to obtain draws from the posterior predictive distribution (only the “answer key” code is shown, the “alternative” code is similar):

```r
model{ for (i in 1:N){ y[i] ~ dpois(lambda) }
   lambda ~ dgamma(alpha,beta)
   x ~ dpois(lambda) # x are the draws from the posterior predictive dsn.
 }
list(y = c(6, 6, 2, 6, 5, 8, 3, 6, 4, 5),N=10,alpha=1,beta=0.001)
```
Mesuaries were calculated using 1000 draws from the posterior predictive distribution \(p(\tilde{y} | y)\).

<table>
<thead>
<tr>
<th>Code</th>
<th>mean</th>
<th>std. dev.</th>
<th>2.5%</th>
<th>Median</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Answer key</td>
<td>5.358</td>
<td>2.472</td>
<td>1</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>Alternative</td>
<td>0.044</td>
<td>0.210</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

The alternative code has such an strong prior that any analysis that we may wish to perform will be affected by this choice of a prior distribution. Note that we have posterior probability close to zero of observing the values of \(y\) that were in fact observed. Thus, there is something wrong with this analysis. This problem is not present on the “answer key” code, the values obtained from the posterior predictive density are very similar to the actual observed values, as one might expect since the prior distribution assigned to \(\lambda\) plays almost no role.

From a computational point of view this tell us that we always have to check whether the program that we are using to carry out our computations does what we expect it to do. Programs are not flexible, you are. So, even though that you may be more familiar with a different parameterization for a given distribution you have to adjust to the parameterization chosen by, in this case, the WinBUGS programmers.