Stat 544 - Spring 2005
Homework 1 answer key

Exercise 1

a- For the sake of simplicity lets replace $F(x)$ by u. Here, u is a draw from an uniform(0,1) distribution.

a.1- Logistic distribution

$$u = \frac{1}{1+\exp\{-(x-\mu)/\tau\}} \Rightarrow x = \mu - \tau \log\left(\frac{1}{u}+1\right)$$

R/S-plus code

```
mu <- 10; tau <- 2
x <- mu - tau * log(1/runif(1000) +1)
```

SAS code

```
%let mu = 10; %let tau = 2;
data sample(drop = i);
     do i = 1 to 1000;
        x = &mu - &tau * log(1/ranuni(-1) +1); output;
     end;
run;
```

a.1- Pareto distribution

$$u = 1 - \left(\frac{b}{x}\right)^a \Rightarrow x = \frac{b}{\sqrt[a]{1-u}}$$

R/S-plus

```
a <- 2; b <- 1
x <- b/(1-runif(1000))^(1/a)  # we did not use the function sqrt()
                              # to make the program more general
```

b- If $X \sim \text{Exp}(\lambda)$ then $T = X^{1/\alpha} \sim \text{Weibull}(\alpha,\lambda)$

c- If $X_i \sim \text{Exp}(1)$ then $Y = \max\{n \mid \sum_{i=1}^{n} X_i \leq \lambda\} \sim \text{Poisson}(\lambda)$.

d- If $X_i \sim \text{Exp}(\beta) = \text{Gamma}(1,\beta)$ then $Y = \sum_{i=1}^{\alpha} X_i \sim \text{Gamma}(\alpha,\beta)$.

e- If $X \sim \text{Gamma}(\alpha,\tau)$ and $Y \sim \text{Gamma}(\beta,\tau)$, then $y/(x+y) \sim \text{Beta}(\alpha,\beta)$.

Thus, to solve parts b through e the only thing that we need is to know how to generate a draw from an exponential distribution given a draw from an uniform(0,1) distribution. This was discussed in class. Given $U \sim \text{U}(0,1)$, $Y = -\log(u)/\lambda \sim \text{Exp}(\lambda)$.

R/S-plus code for part b

```
lambda <- 3; alpha <- 2
x <- (-log(runif(1000)/lambda)^alpha
```

R/S-plus code for part c

```
x <- NULL; lambda <- 5
for (i in 1:1000) {u <- 0; n <- 0
                   while (u < lambda) {u=u-log(runif(1)); n=n+1}
                   x[i] <- n-1 #the while loop will iterate one
                               #time more than needed
                  }
```

R/S-plus code for part d

```
x <- NULL; beta <- 3; alpha <- 2
for (i in 1:1000) x[i] <- sum(-log(runif(alpha))/beta)
```

R/S-plus code for part e

```
x1 <- x2 <- NULL; beta <- 5; alpha <- 3
for (i in 1:1000){ x1[i] <- sum(-log(runif(alpha)))
                   x2[i] <- sum(-log(runif(beta))) }
x <- x2/(x2+x1)
```

Exercise 2

The WinBugs programs needed to solve this exercise are listed below. No numerical values are provided.

1- This part was solved in the mini-tutorial.

2-
```
model{ for (i in 1:N){ y[i] ~ dbin(p,n) }
         p ~ dbeta(alpha,beta)
       }

list(y = c(6, 5, 6, 7, 7, 4, 5, 8, 6, 9),N=10,n=10,alpha=0.5,beta=0.5)
```

Note that N and n are two different objects. Here N was chosen to represent the total of observed values of y and n represents the parameter of the binomial distribution that is considered fixed. Advise: try to avoid using this feature.

3-
```
model{ for (i in 1:N){ y[i] ~ dpois(lambda) }
         lambda ~ dgamma(alpha,beta)
       }
list(y = c(6, 6, 2, 6, 5, 8, 3, 6, 4, 5),N=10,alpha=1,beta=0.001)
```

Exercise 3

Prize is in box	Contestant chooses box	Host opens box	Contestant switches	Result
A	A	B or C	A for B or C	loses
A	B	C	B for A	wins
A	C	B	C for A	wins
B	A	C	A for B	wins
B	B	A or C	B for A or C	loses
B	C	A	C for B	wins
C	A	B	A for C	wins
C	B	A	B for C	wins
C	C	A or B	C for A or B	loses

So, enumeration shows that probability of winning by switching is $6/9 = 2/3$.

Exercise 4

1- $P(.4 < \theta < .6) = P(.4 < \theta < .5) + P(.5 < \theta < .6) = 0.498 + 0.498 = 0.996$.

The fact that $P(.4 < \theta < .5) = P(.5 < \theta < .6)$ represents our ambivalence respect to whether θ is greater than 0.5.

2- $X_1, \ldots, X_{1000}$ iid r.v. such that $X_i \sim \text{Bernoulli}(\theta)$, $i = 1, \ldots, 1000$. Define $X = (X_1, \ldots, X_{1000})$ then

$$p(x|\theta) = \prod_1^{1000} \theta_i^x (1-\theta)^{1-x_i} = \theta^{\sum x_i}(1-\theta)^{1000-\sum x_i}$$

Thus,

$$p(\theta|X) \propto \theta^{99+\sum x_i}(1-\theta)^{1099-\sum x_i}$$

Then, $\theta|X \sim \text{Beta}(100 + \sum x_i, 1100 - \sum x_i)$. For this problem $\sum x_i = 489$. Then $\theta|X \sim \text{Beta}(589, 611)$, which gives the result $P(\theta > 0.5) = 0.26261$